Worksheet- Set theory   **Name:**
Part 1: 11/08/2024

Notations:
$\mathbb{N}$ is the set of all natural numbers, $\{1, 2, 3, 4, \cdots\}$,
$\mathbb{Z}$ is the set of all integers, $\{\cdots, -2, -1, 0, 1, 2, \cdots\}$.
$\mathbb{R}$ is the set of all real numbers.

1. What are the elements of the following sets? Describe explicitly.

   (a) The set of people with 10 heads.
   (b) $A = \{n \in \mathbb{N} : n$ is an even prime number$\}$
   (c) $B_1 = \{x \in \mathbb{N} : x^2 = 4\}$
   (d) $B_2 = \{x \in \mathbb{Z} : x^2 = 4\}$
   (e) $\{x \in \mathbb{R} : x < 0\}$.
   (f) $\{x \in \mathbb{R} : x(x-1)(x-2) < 0\}$.

2. What must you do to prove that $\mathbb{N} \subsetneq \mathbb{Z}$?

3. Are the following sets equal?

   (a) $A = \{h : h$ is human being with 5 legs$\}$, $B = \{x \in \mathbb{R} : x^2 + 1 = 0\}$
   (b) $C = \{7m + 1 : m \in \mathbb{Z}\}$, $D = \{7n - 6 : n \in \mathbb{Z}\}$.

4.* Let $A = \{x \in \mathbb{R} : x^2 > x + 6\}$ and $B = \{x \in \mathbb{R} : x > 3\}$. Which of the following is true? (i) $A \subseteq B$, (ii) $B \subseteq A$?

5. Prove that $\varnothing \subset A$, for any set A. What is $A \cup \varnothing$? $A \cap \varnothing$?

6. Let $A = \{\varnothing, \{\varnothing\}\}$. Is $\varnothing \in A$? How about $\{\varnothing\}$? Is $\{\varnothing\} \subset A$? Lastly, is $\{\{\varnothing\}\} \in A$ or $\{\{\varnothing\}\} \subset A$?

7. Let S be a subset of $\mathbb{N}$ with the property that if a positive integer $n \in S$, then $n + 1 \in S$. Describe all such such subsets of S.

8.* Let $\Omega = \mathbb{N}$, $A = \{1, 2, 3, 4\}$, and $B = \{3, 4, 5, 6\}$. What is $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$, and $A \Delta B$?

9. Prove that $A \cap B \subseteq B$. Prove that $A \cap B = B$ if and only if $B \subseteq A$.

10.* Prove that $A \cup B \supseteq B$. Prove that $A \cup B = B$ if and only if $B \supseteq A$.

11. What is $(A^c)^c$?

12. Prove $A \setminus B = A \cap B^c$. How about $(A \setminus B)^c$?

13. **De Morgan's Laws:**

    (a) $(A \cup B)^c = A^c \cap B^c$.
    (b) $(A \cap B)^c = ?$