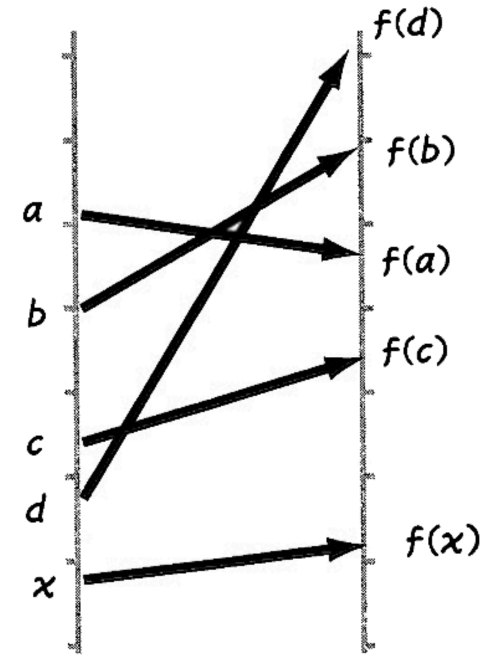
FUNCTIONS

DEFINITION OF A SET

- A set is a well-determined collection of distinct objects that satisfies the ZFC conditions
- Well-determined refers to a specific property which makes it possible to identify whether a given object belongs to a set or not
- Zermelo-Fraenkel set theory with choice (ZFC) conditions
  - Does not allow a set corresponding to every property
  - Does not allow a set to contain itself
  - Does not allow a set containing all sets

FUNCTIONS

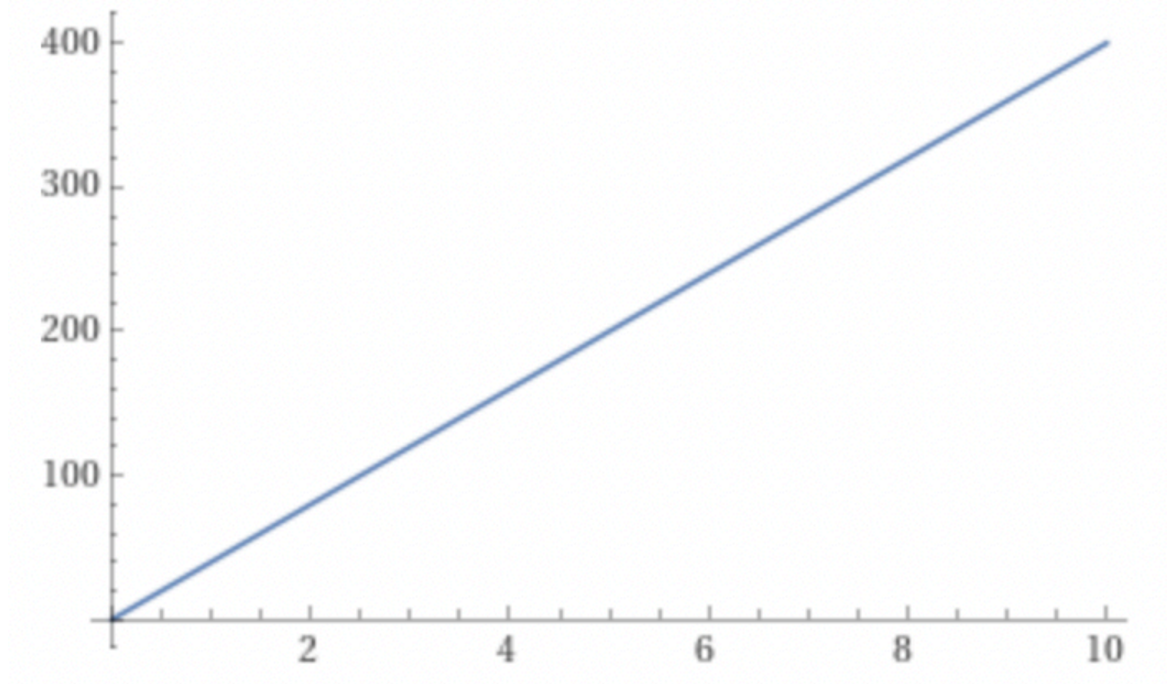
- A function $f: X \to Y$ from a set X to a set Y assigns to each element $x \in X$ exactly one element $y \in Y$ (denoted by $f(x) = y$)
- X is called domain of f and Y is called the codomain of f
- Range of f is $\{y \in Y : \exists x \in X, f(x) = y\}$



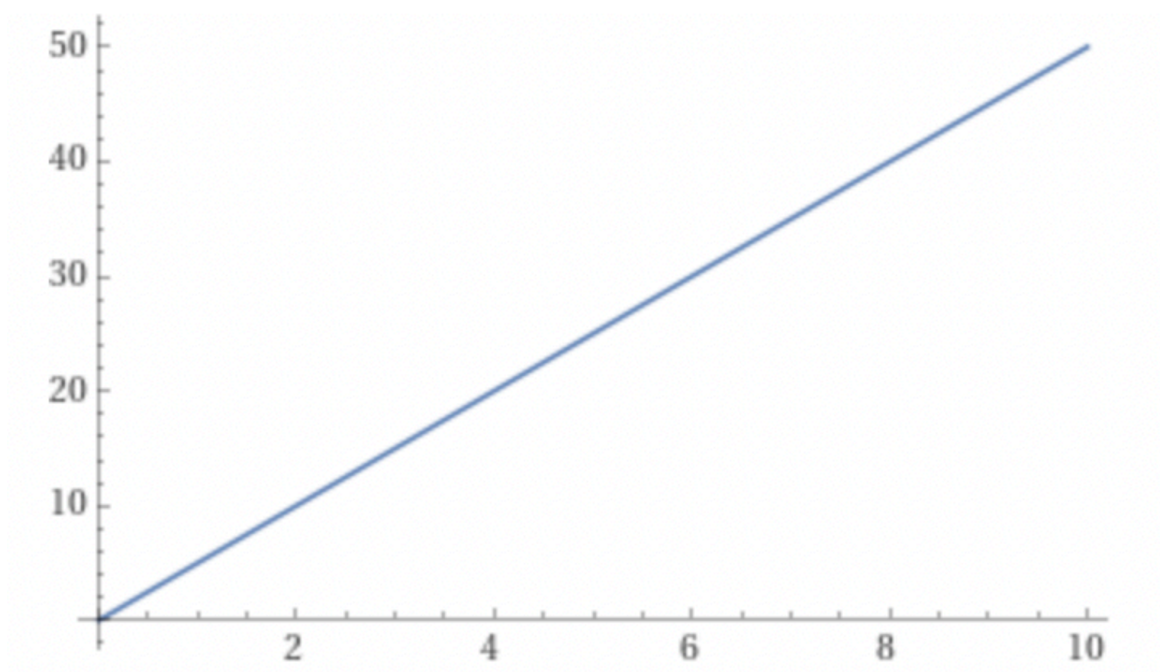
- Two functions f and g are equal if they have the same domain X, same codomain Y and for each $x \in X, f(x) = g(x)$

FUNCTIONS

- Car travels at a constant speed of 40kmh^{-1}. Write down a function that tells you the distance traveled given the total travel time.
  - $d(t) = 40 * t$

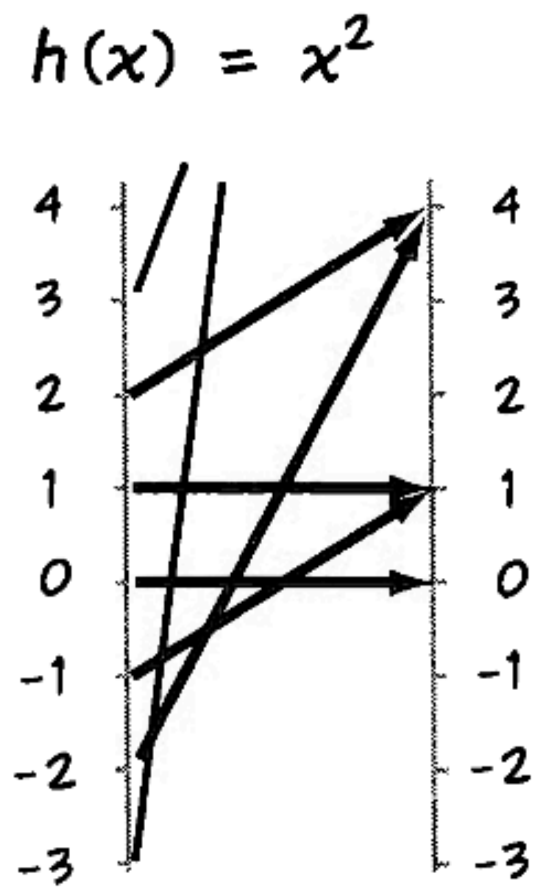


- The IIT Palakkad library charges Rs. 5 per day for an overdue book. Write a function that takes as input "the no of days overdue" and gives as output the amount to be paid as a fine.
  - $f(d) = 5 * d$

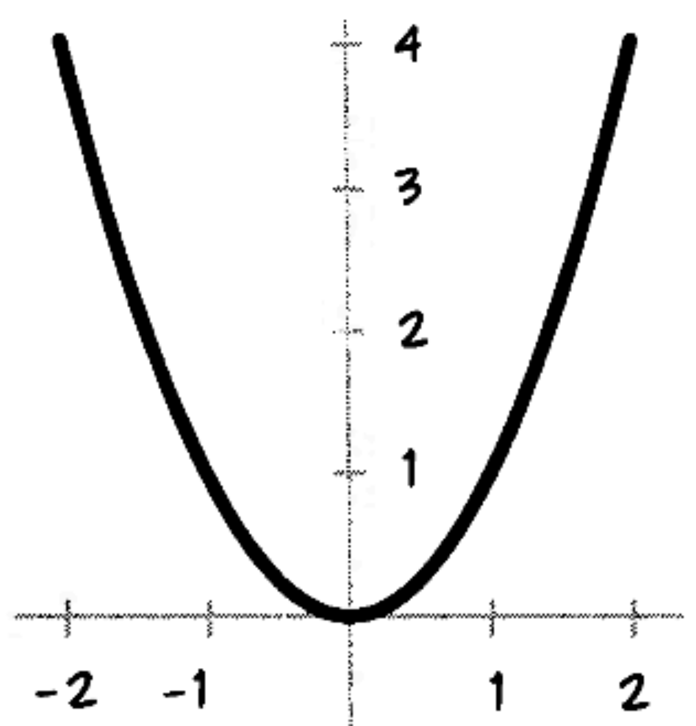


REAL FUNCTIONS

- Real-valued function of a real variable $f : \mathbb{R} \to \mathbb{R}$
- Assigns to each $x \in \mathbb{R}$ exactly one element $y \in \mathbb{R}$ (denoted by $f(x) = y$)



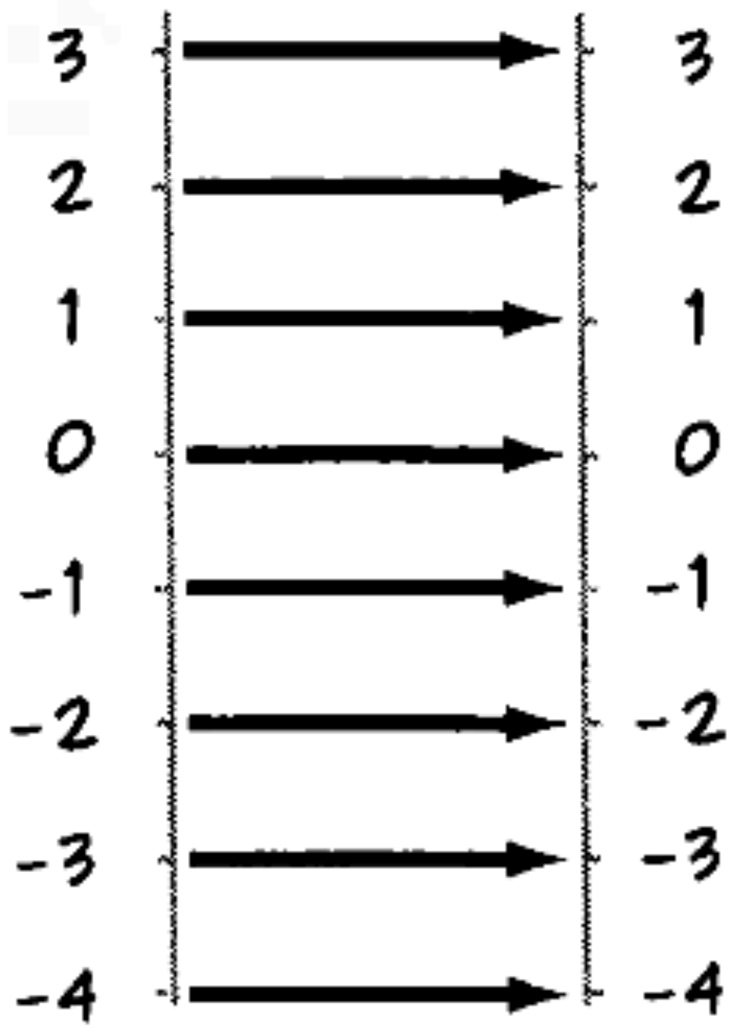
ARROWS



GRAPH

FUNCTIONS

$$f(x) = x$$



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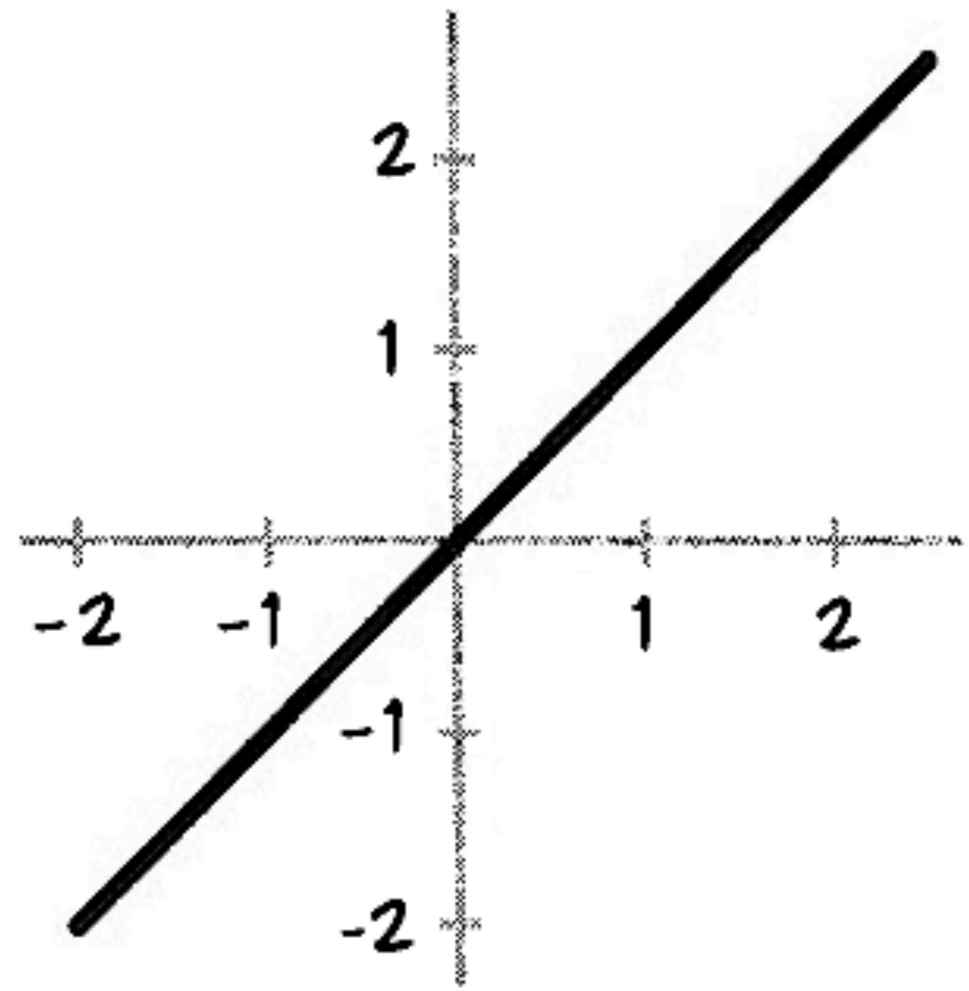
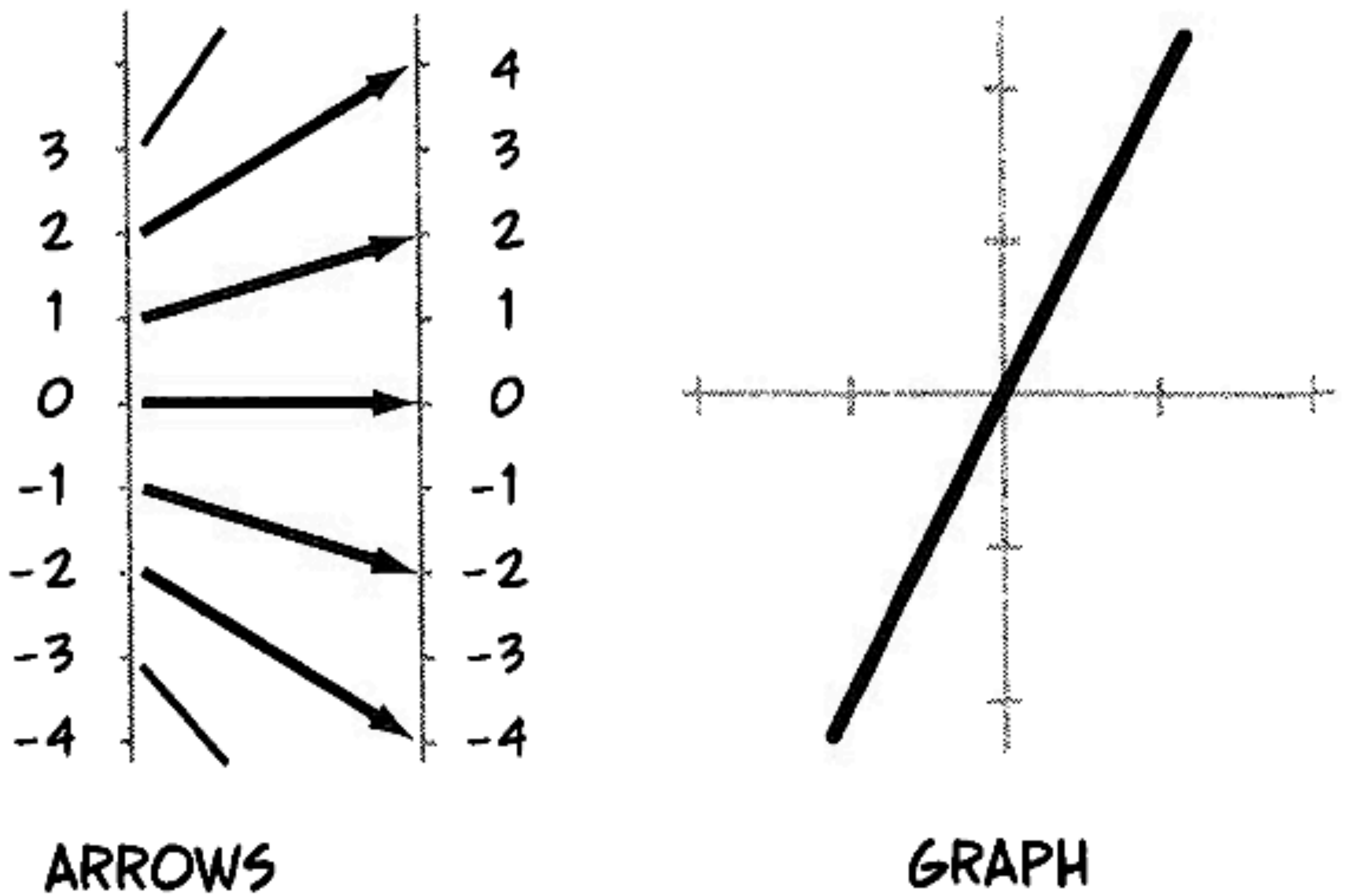


GRAPH

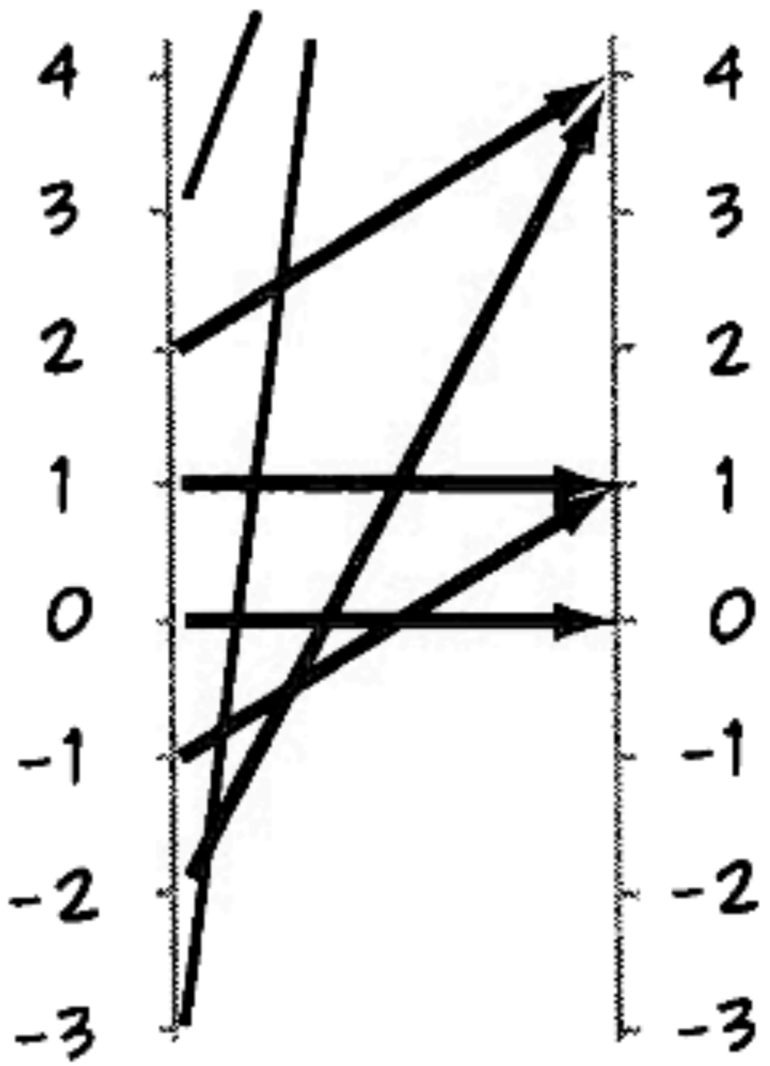
FUNCTIONS

$$g(x) = 2x$$

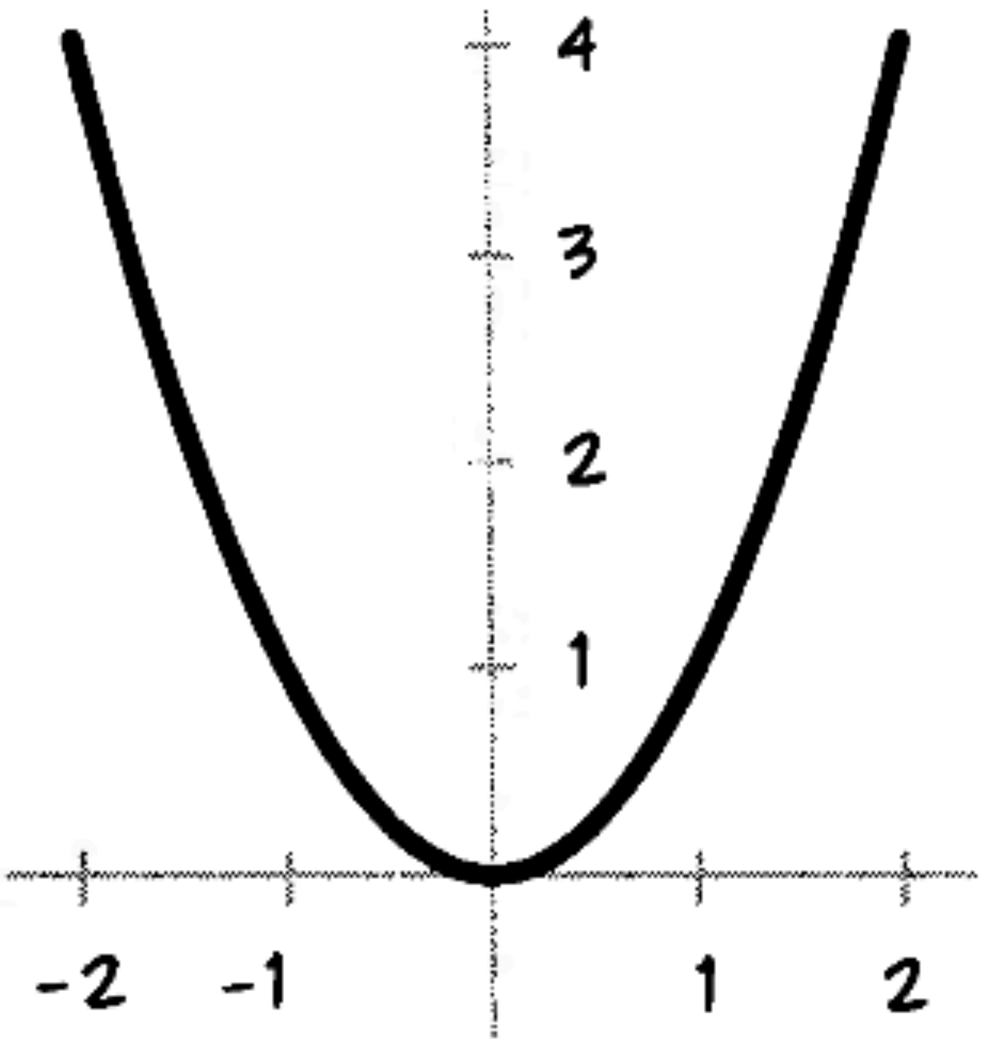


FUNCTIONS

$h(x) = x^2$

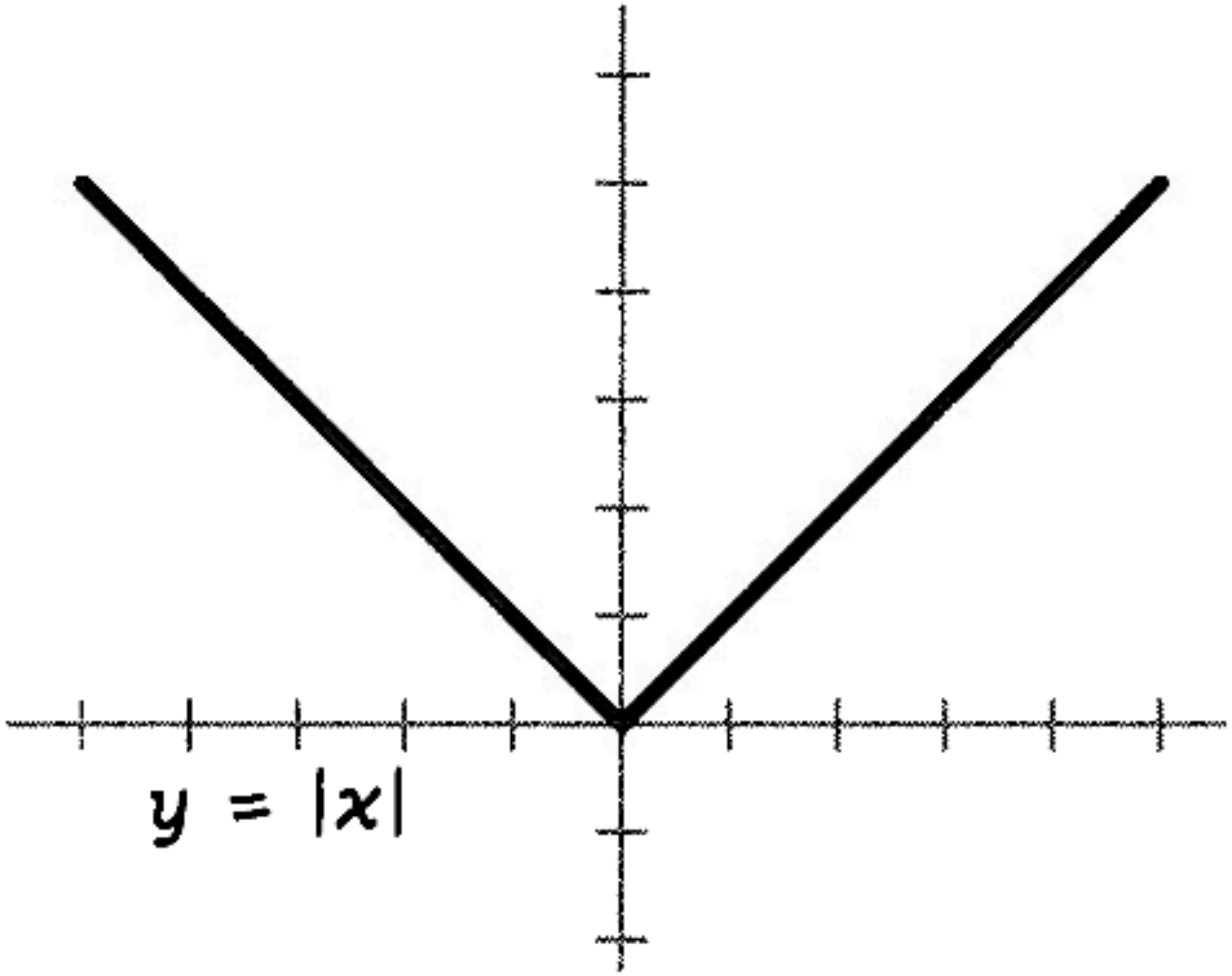


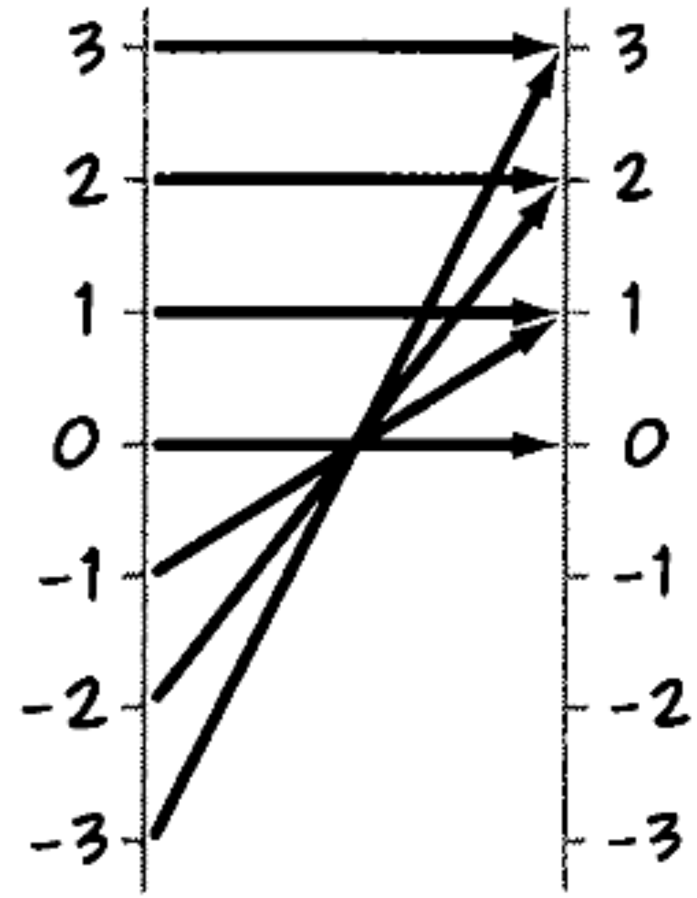
ARROWS



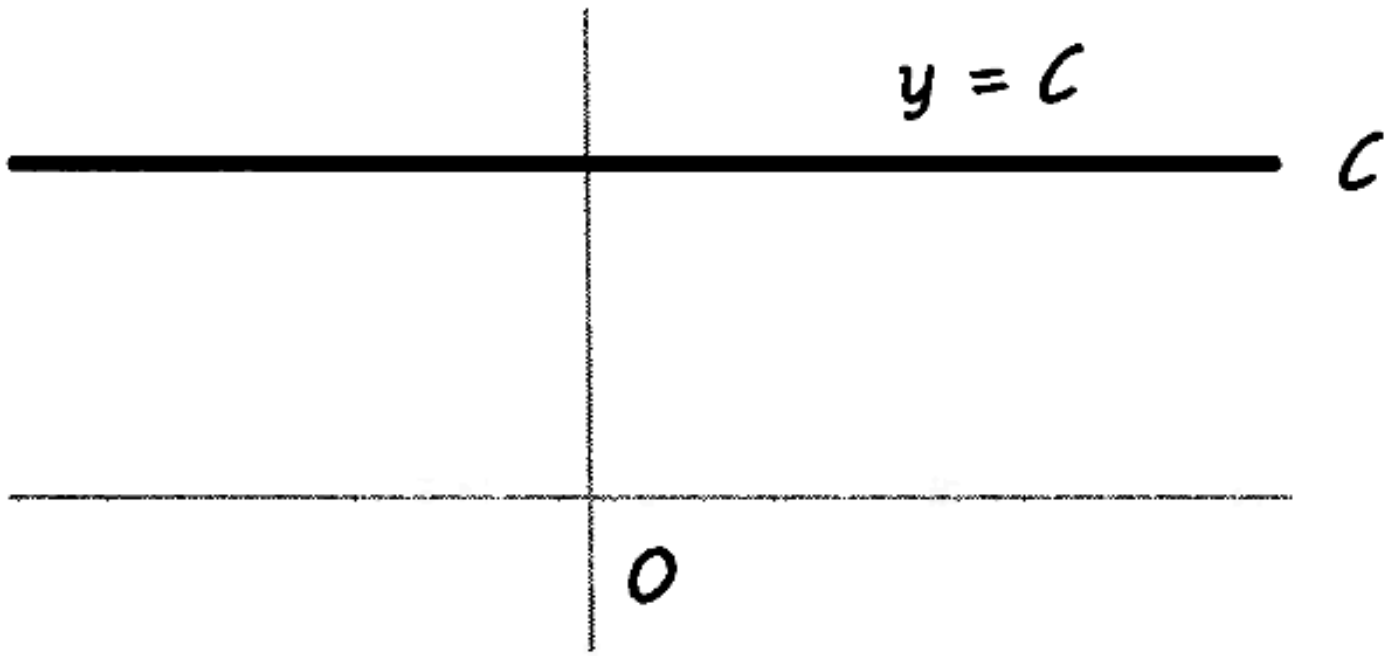
GRAPH

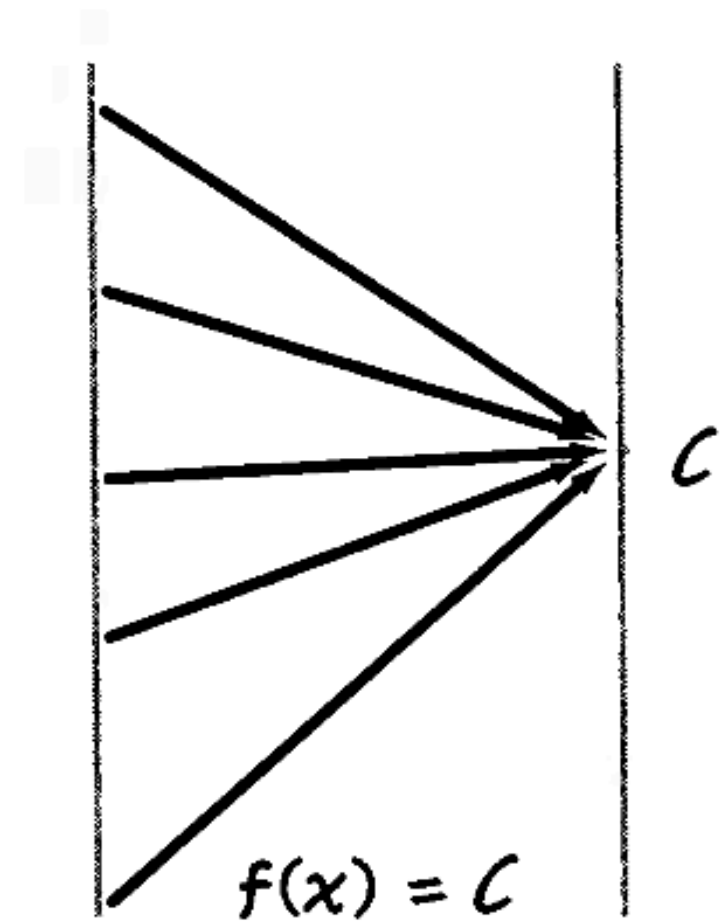
FUNCTIONS



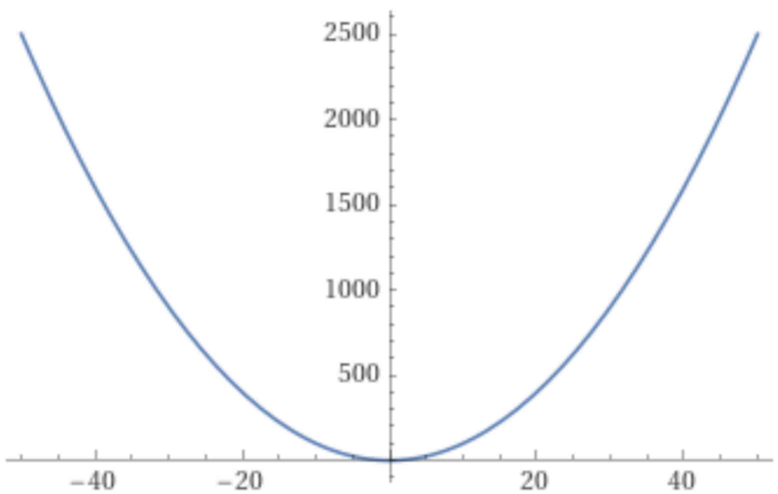


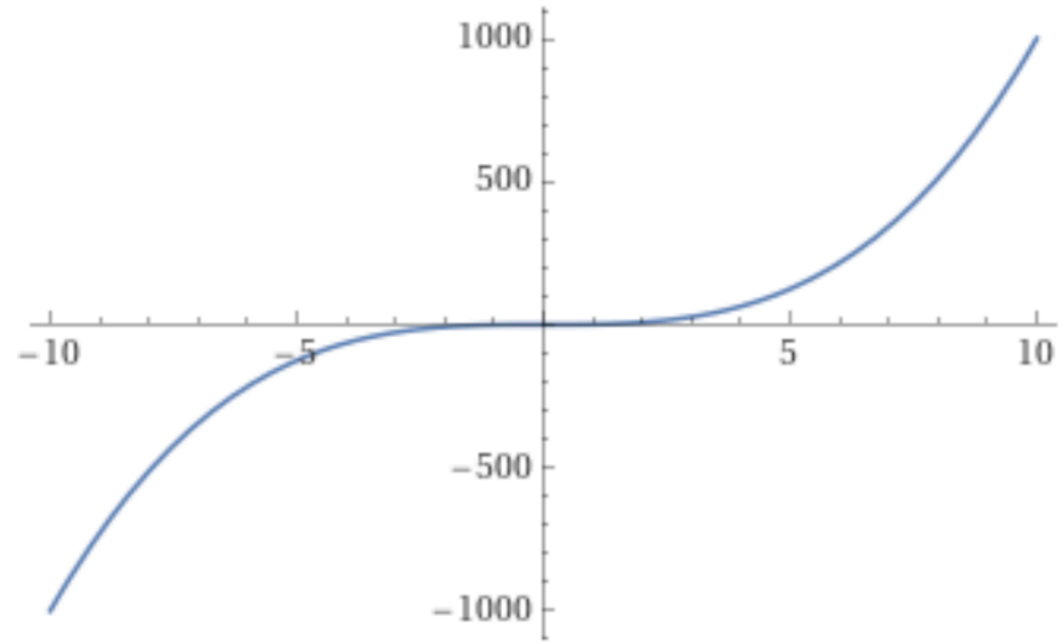
FUNCTIONS

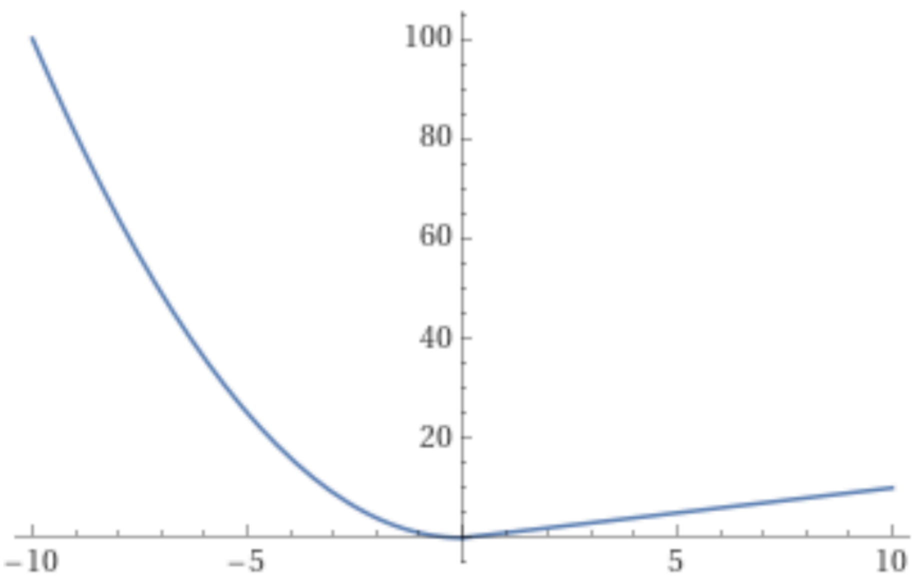


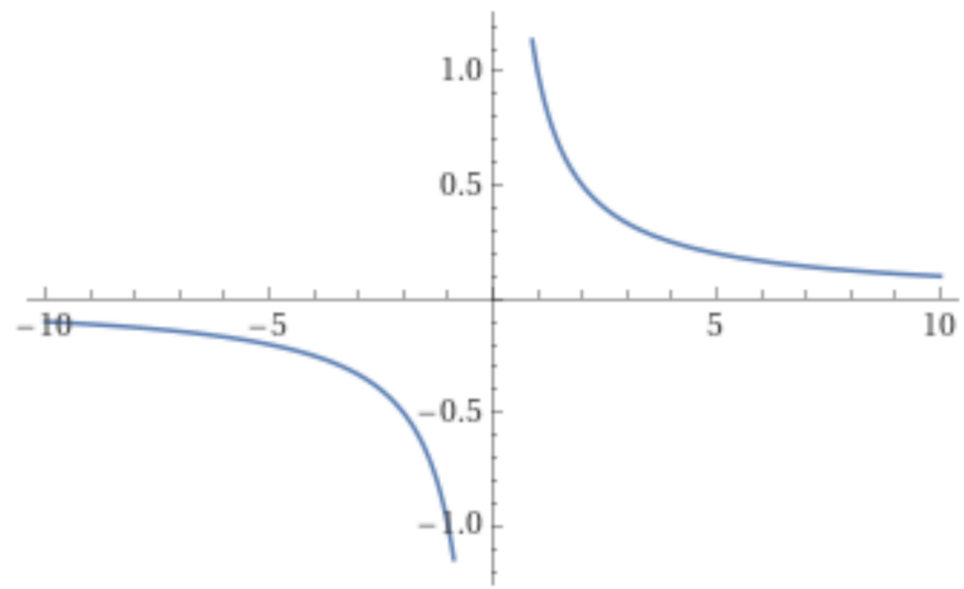


FUNCTIONS



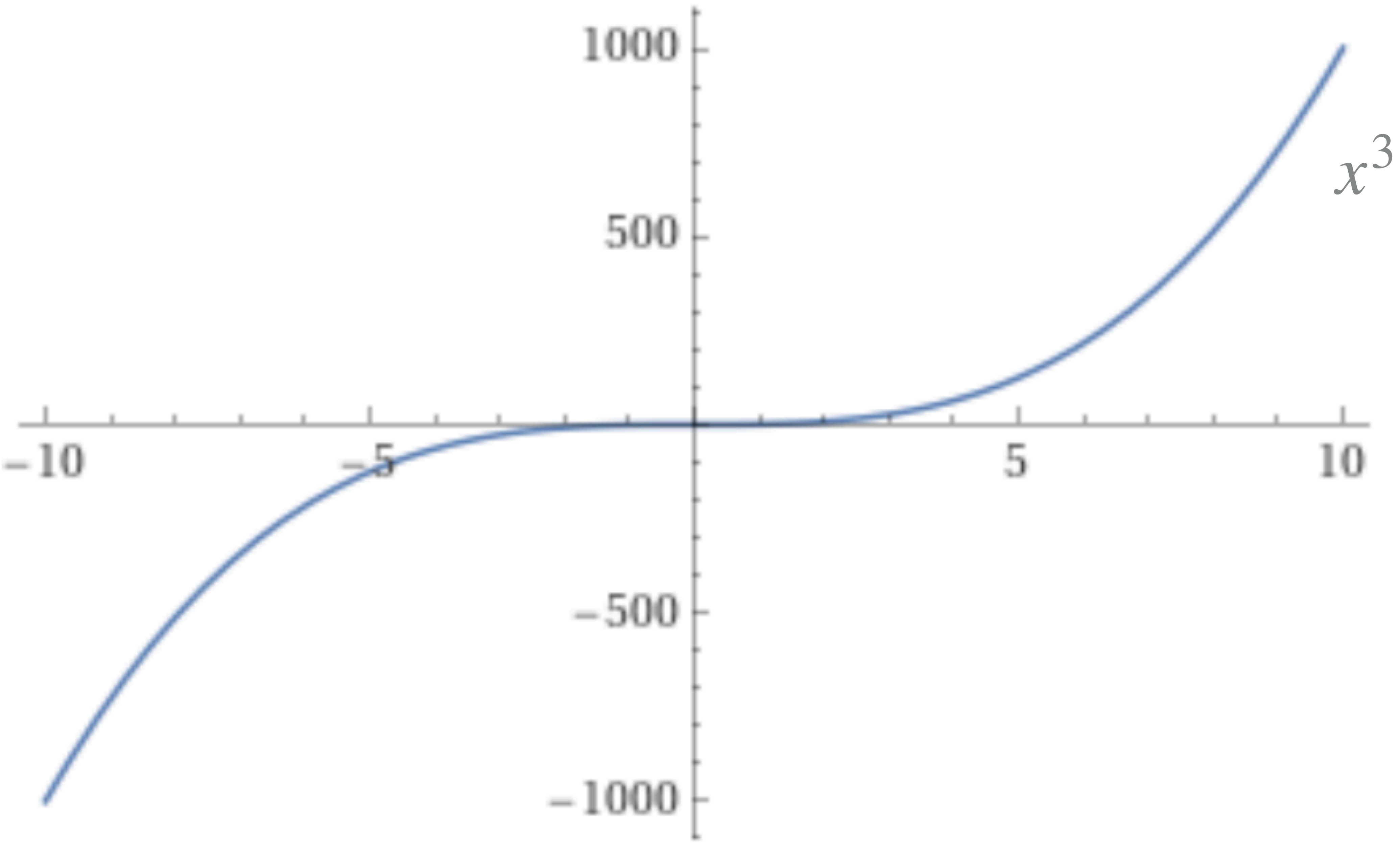




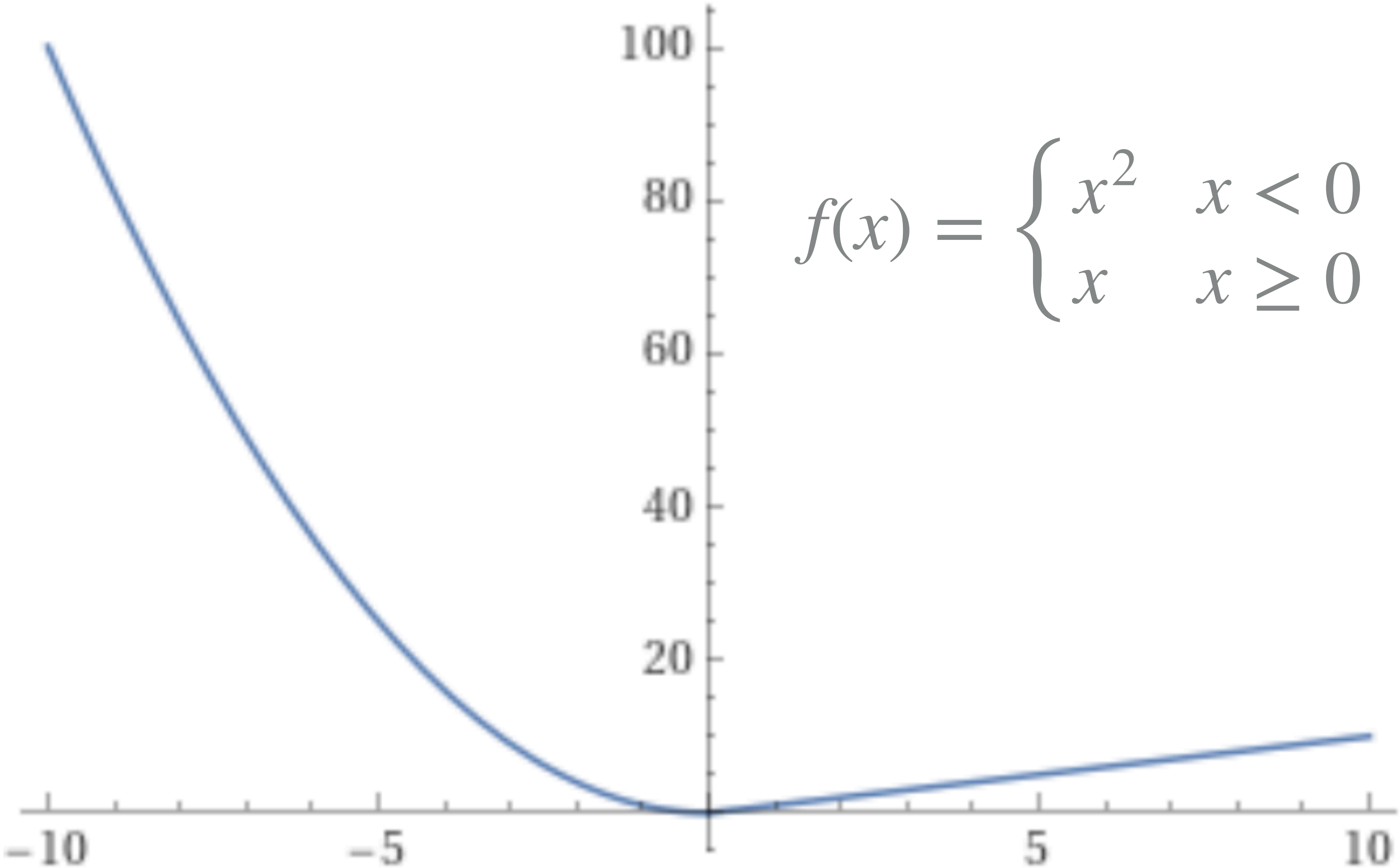


- How do we plot the function when the domain is infinite?

FUNCTIONS

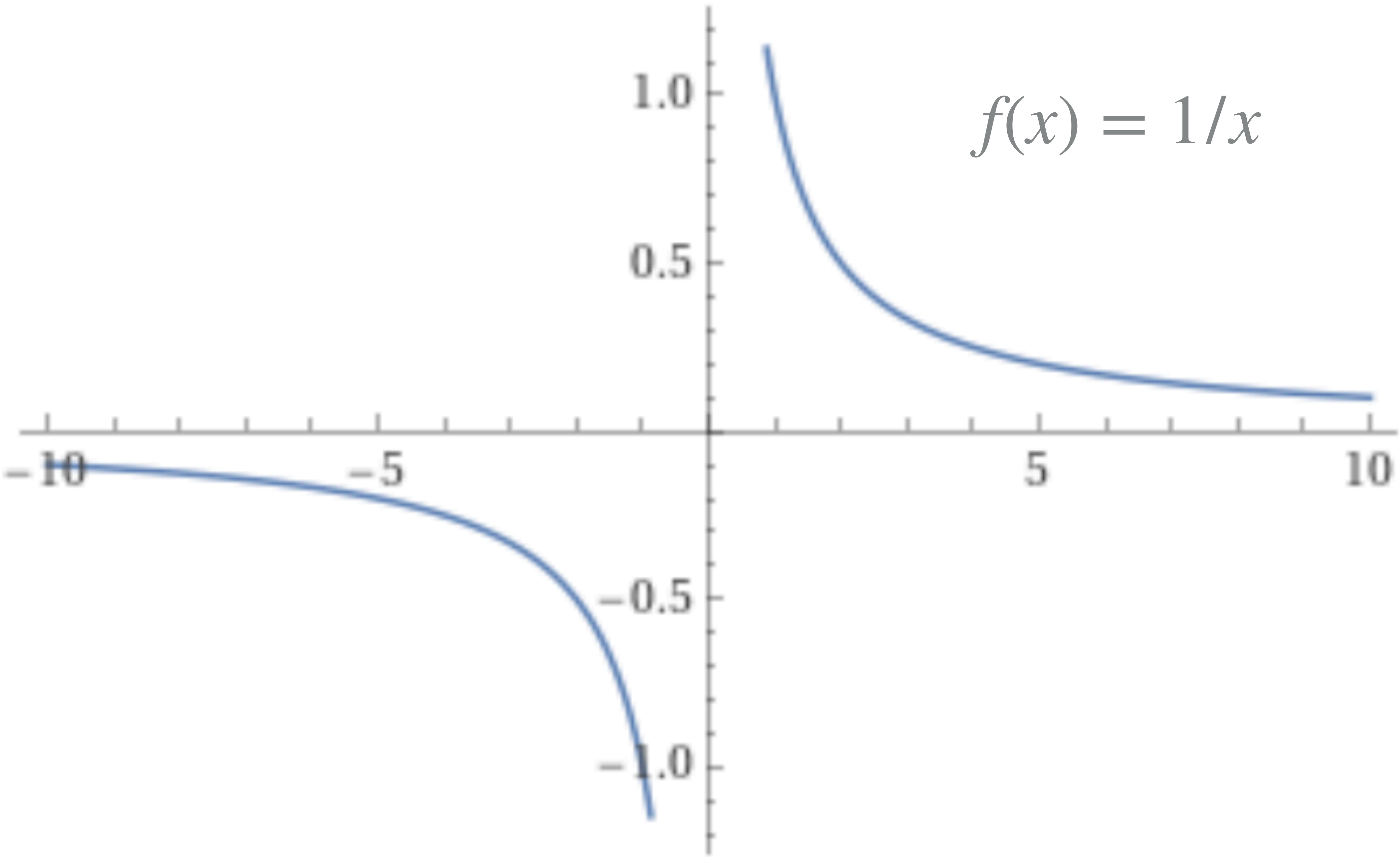


FUNCTIONS



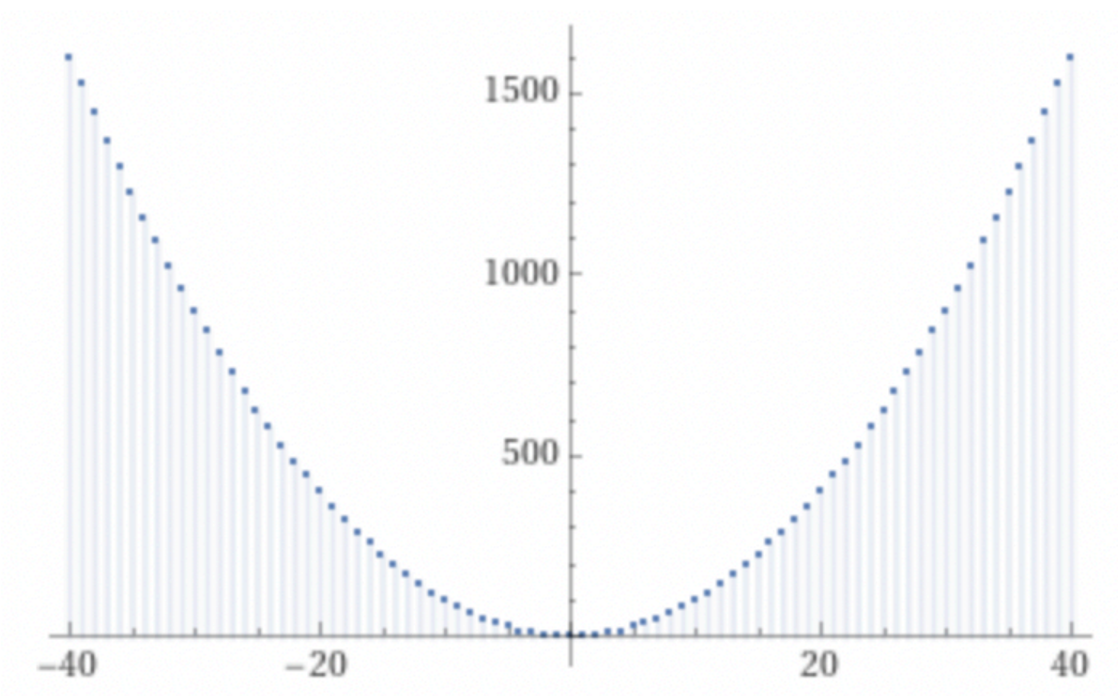
$$f(x) = \begin{cases} x^2 & x < 0 \\ x & x \geq 0 \end{cases}$$

FUNCTIONS

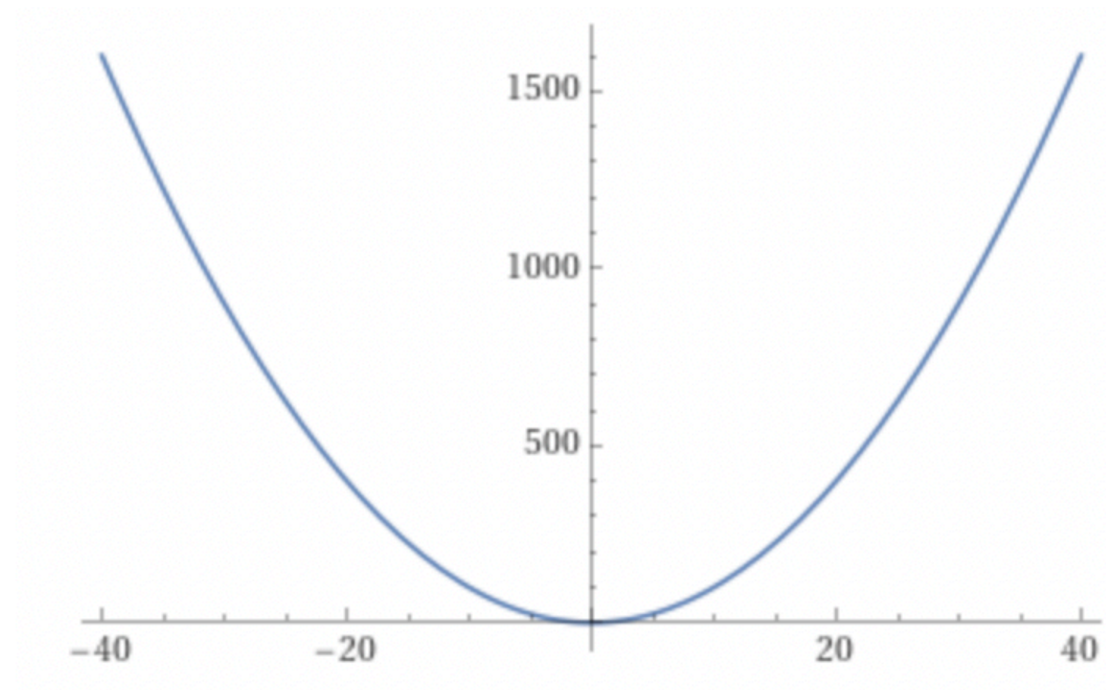


- What is the domain of this function?

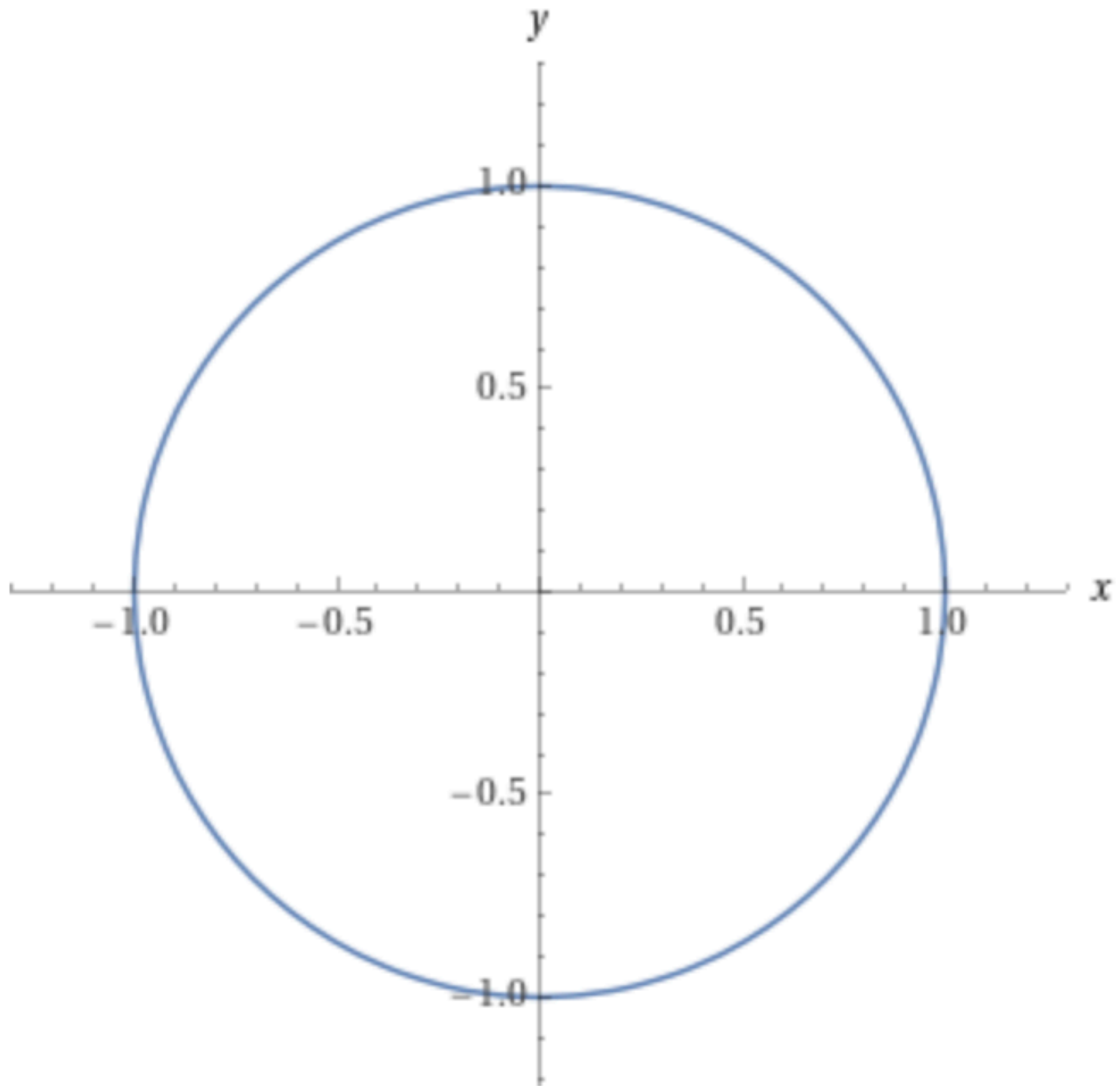
FUNCTIONS



x^2



FUNCTIONS

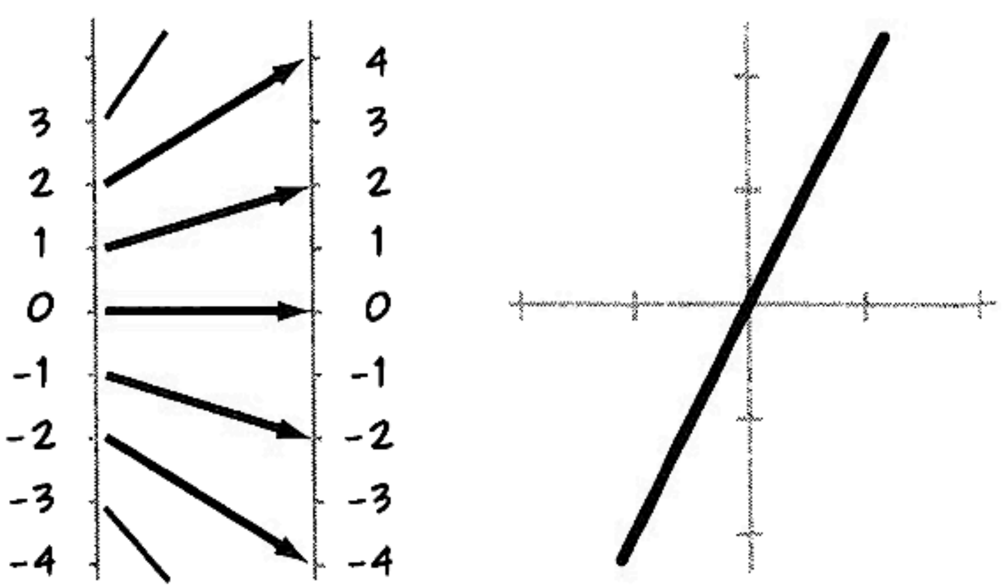


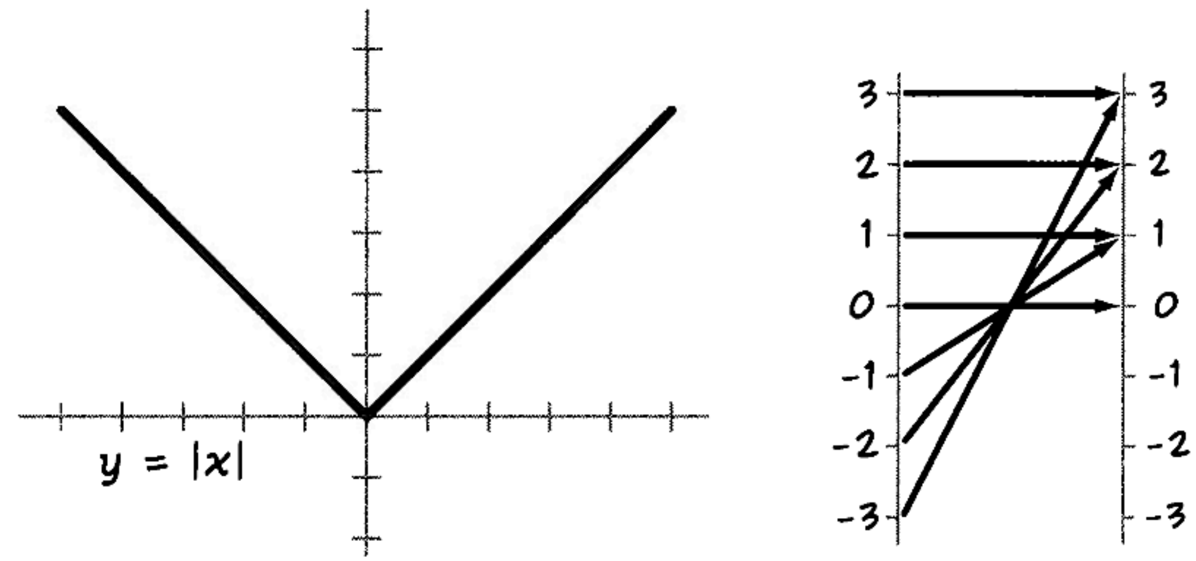
- Is this a function? Do a vertical-line test!

INJECTIONS

- A function $f : X \to Y$ is an injection (or a one-to-one function) if for each $x, x' \in X$ such that $x \neq x'$ then $f(x) \neq f(x')$

g(x) = 2x

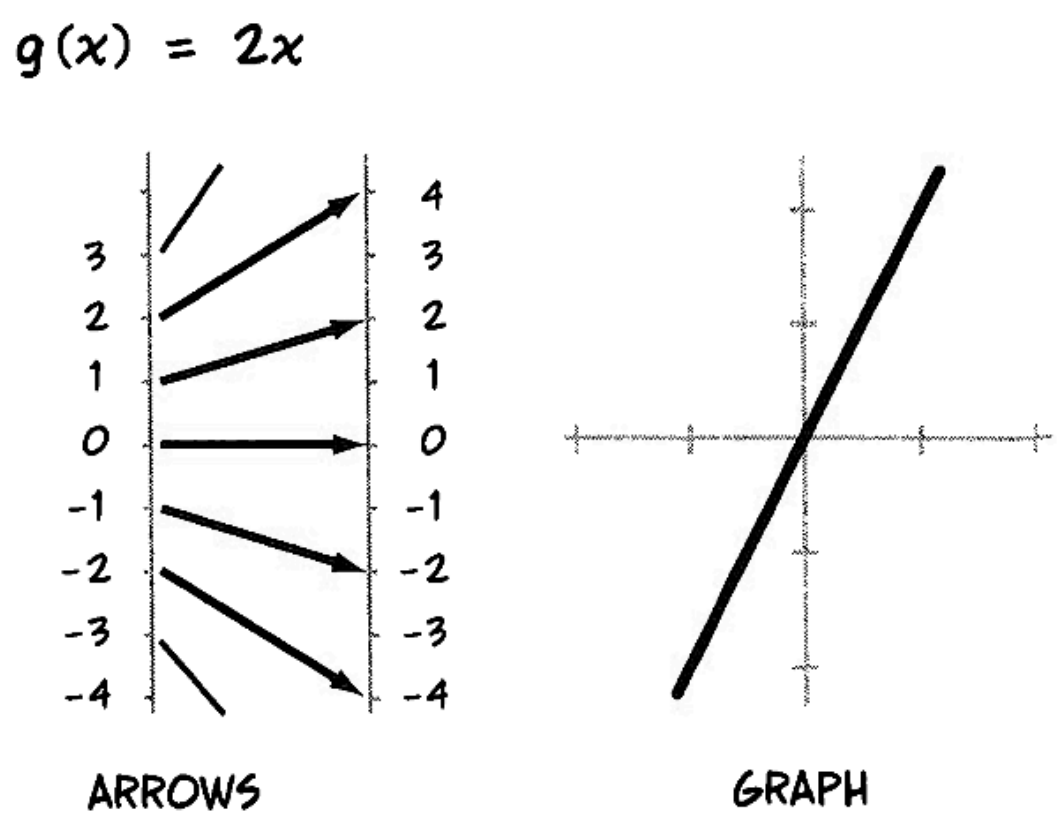


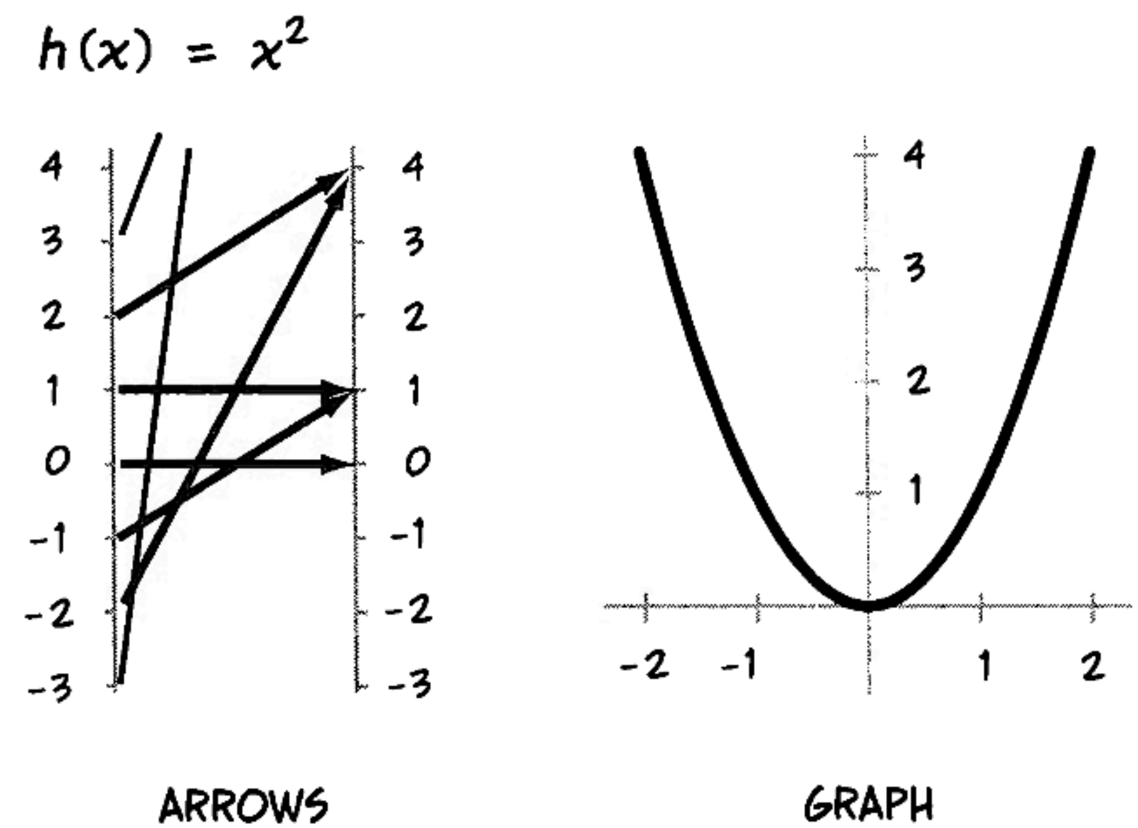


- Horizontal line test: every line parallel to x-axis intersects with the function in at most one point

SURJECTIONS

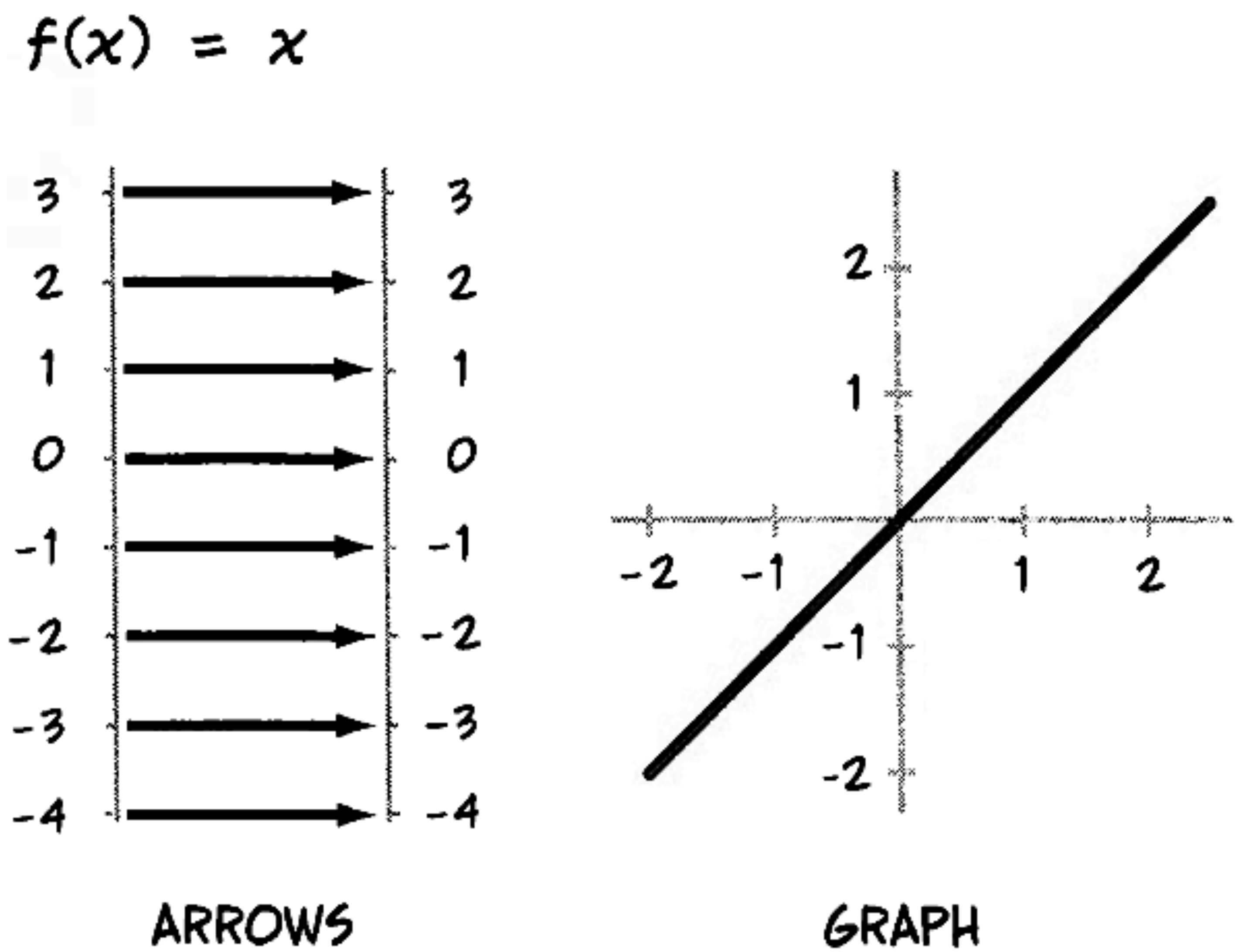
- A function $f : X \to Y$ is a surjection (or an onto function) if for each $y \in Y$ there is $x \in X$ such that $f(x) = y$



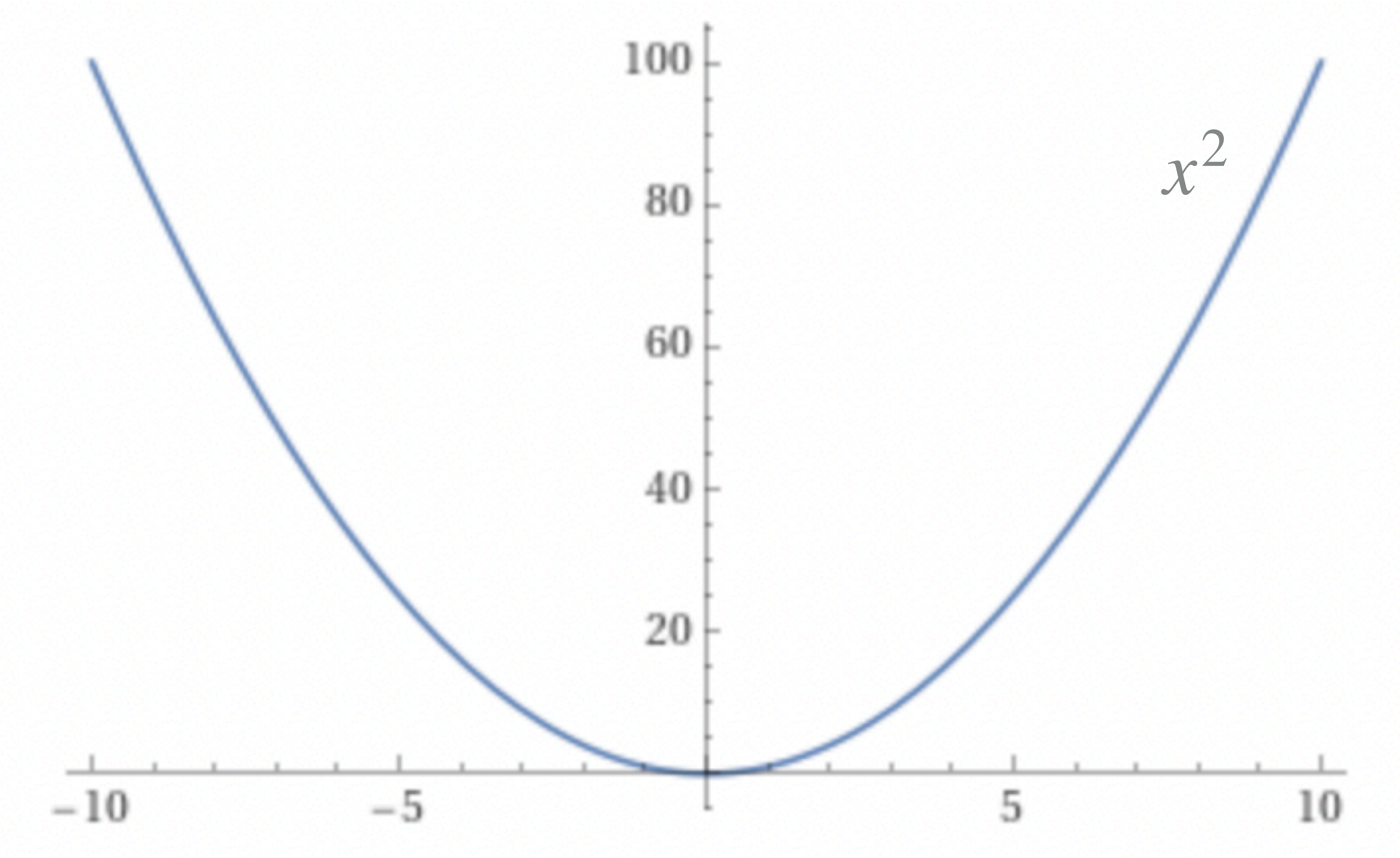


BIJECTIONS

- A function $f : X \to Y$ is a bijection if it is an injection and a surjection

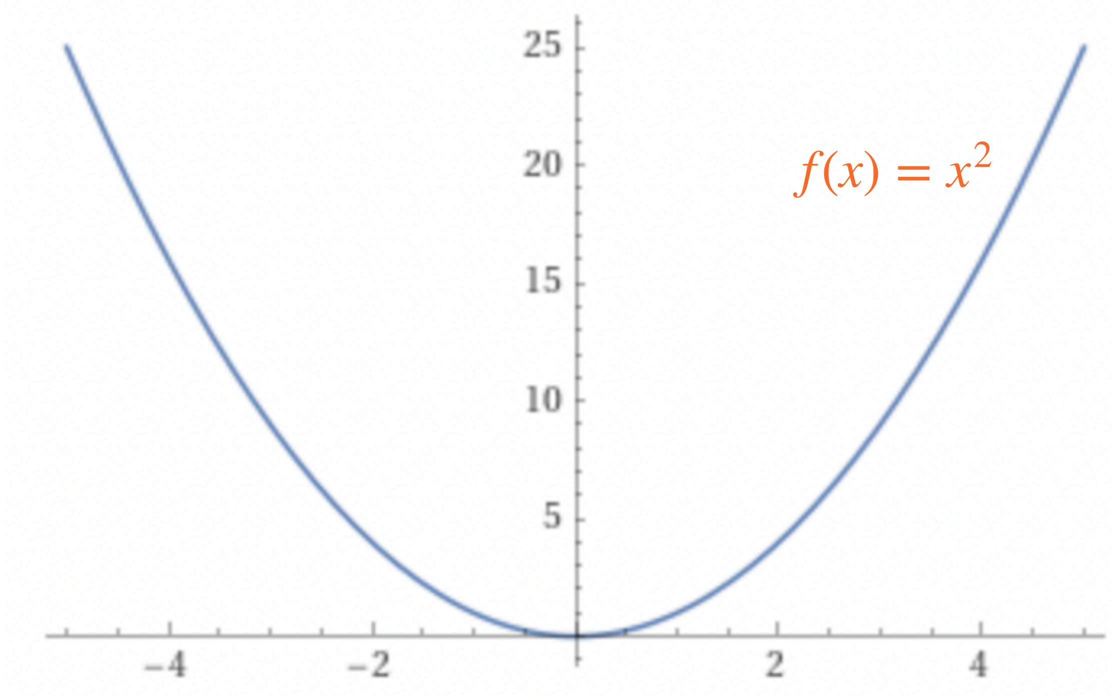


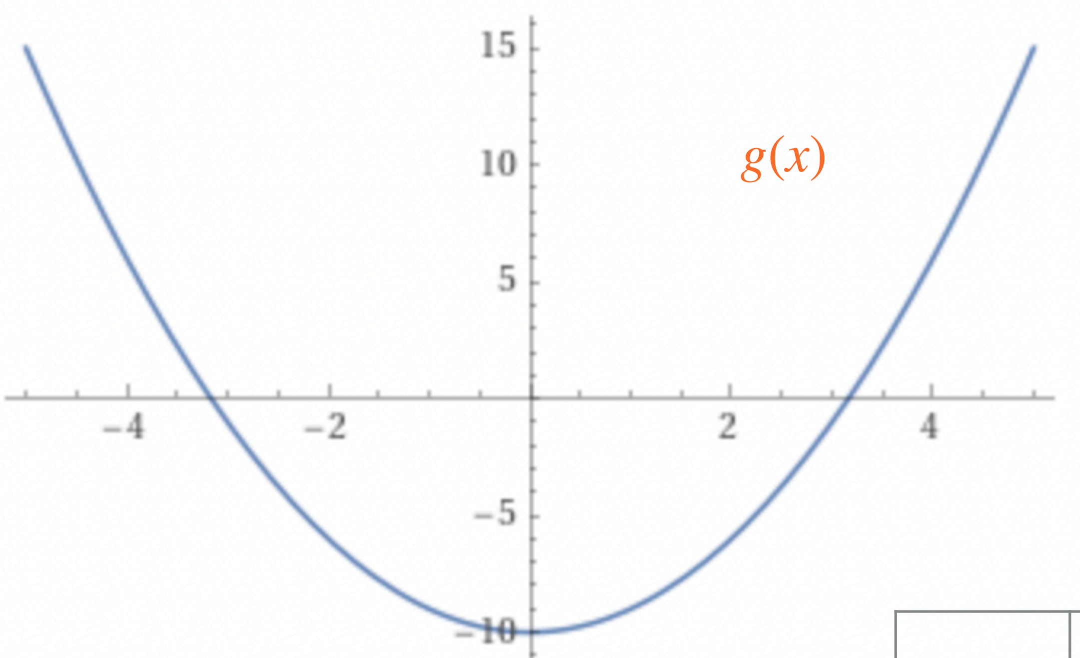
$f: \mathbb{R} \to \mathbb{R}$, where $f(x) = x^2$ for each $x \in \mathbb{R}$

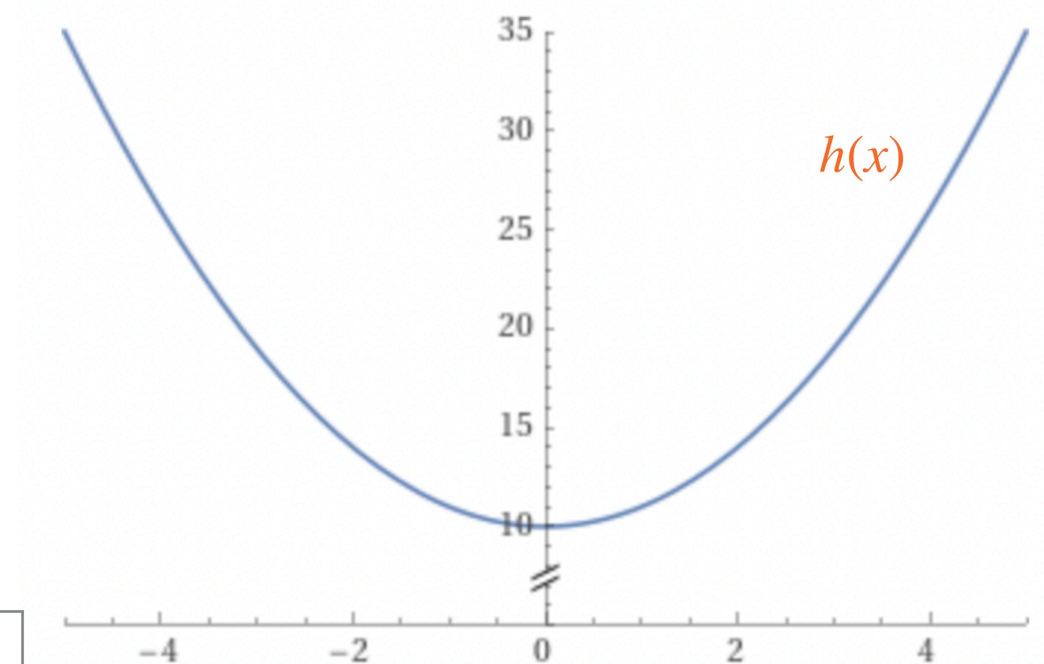


How to move the function up or down?

Change the point of intersection of function with y-axis

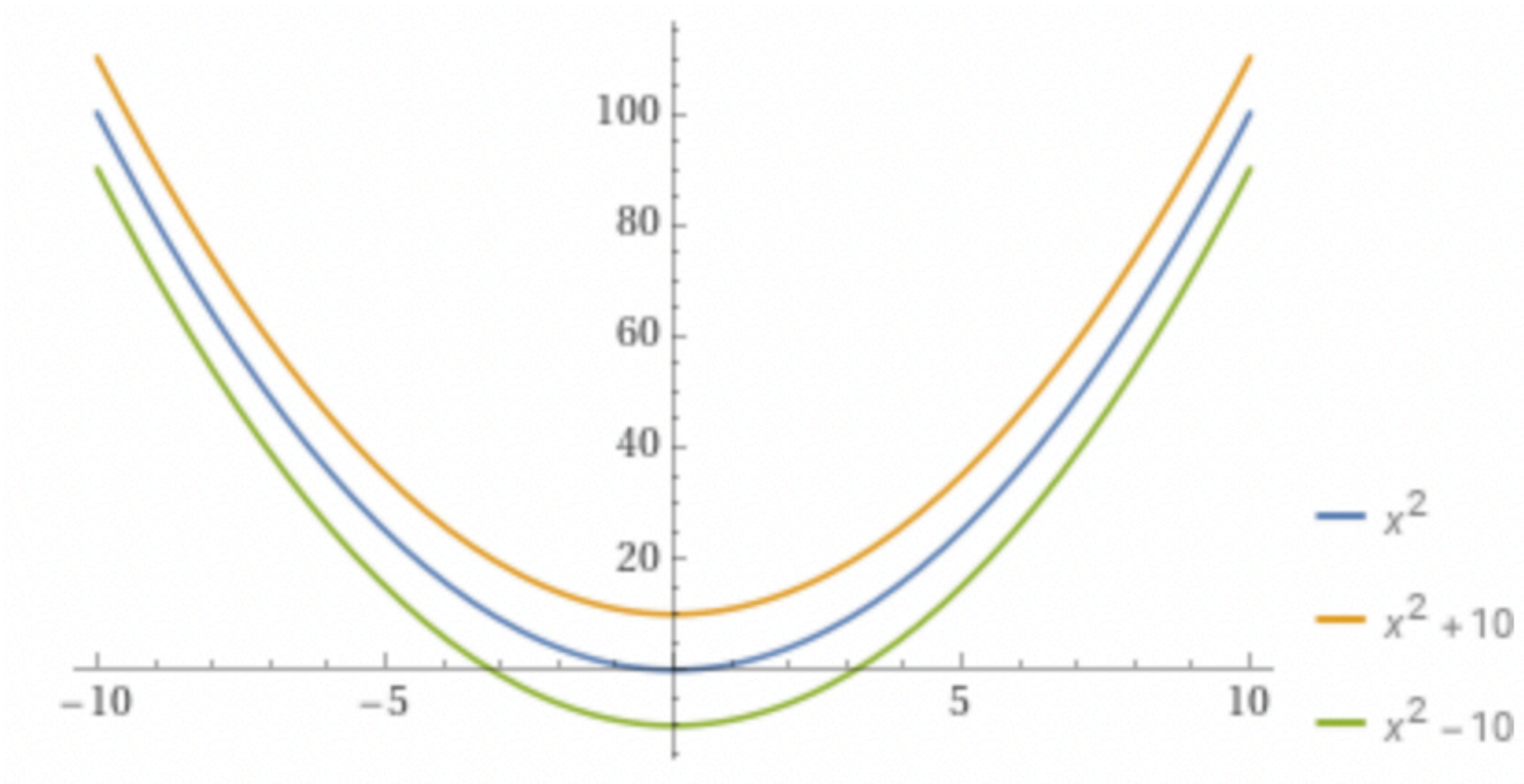






	$f(x)$	$g(x)$	$h(x)$
$x = 0$	0	-10	10
$x = 1$	1	-9	11
$x = 2$	4	-6	14
$x = 3$	9	-1	19

TRANSFORMATION – VERTICAL TRANSLATION



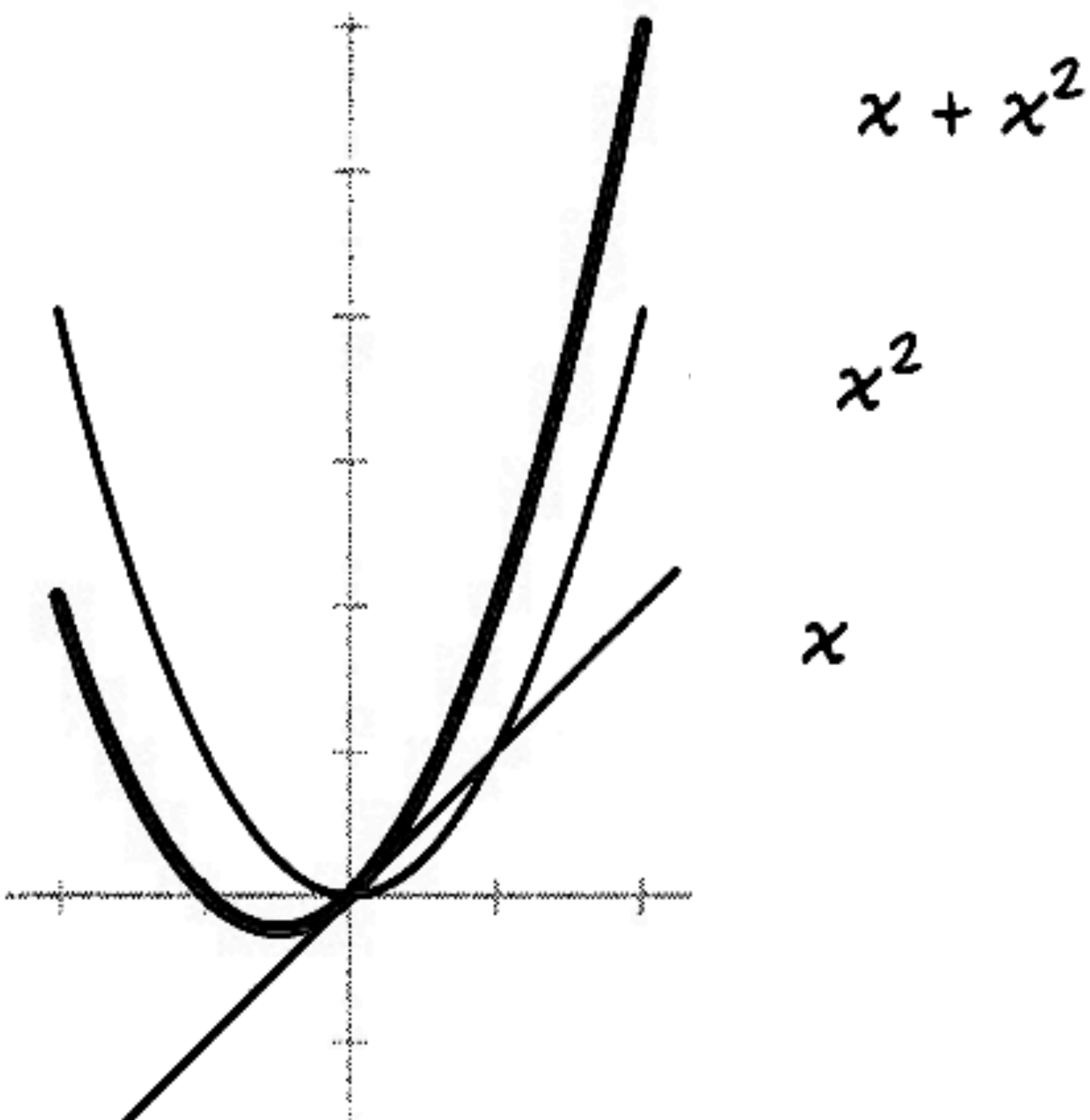
Move f up: $h(x) = f(x) + 10$

Move f down: $g(x) = f(x) - 10$

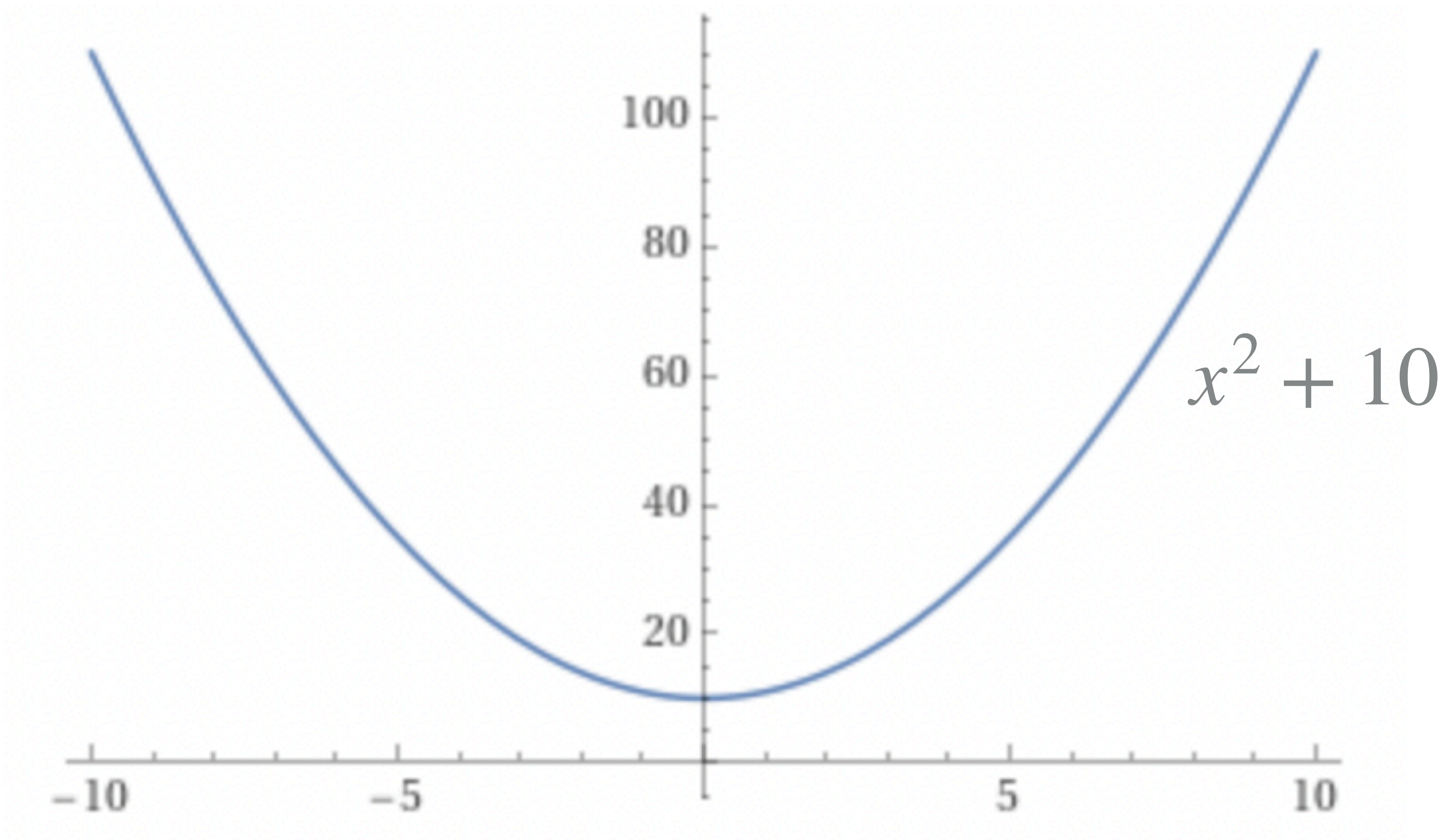
- h is the new function obtained from f
  - For each x, add 10 to $f(x)$
- g is the new function obtained from f
  - For each x, subtract 10 from $f(x)$

COMBINING FUNCTIONS

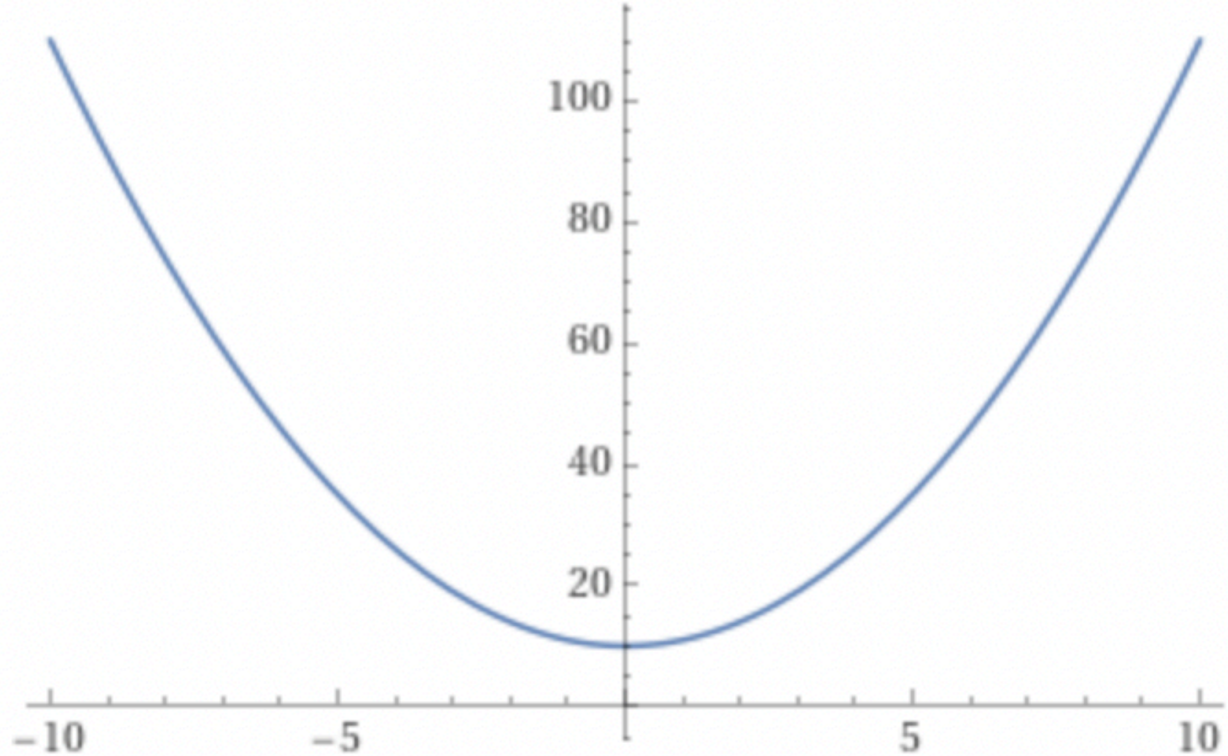
- For functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$, define two new functions
  - Sum $(f + g)(x) = f(x) + g(x)$
  - Difference $(f - g)(x) = f(x) - g(x)$

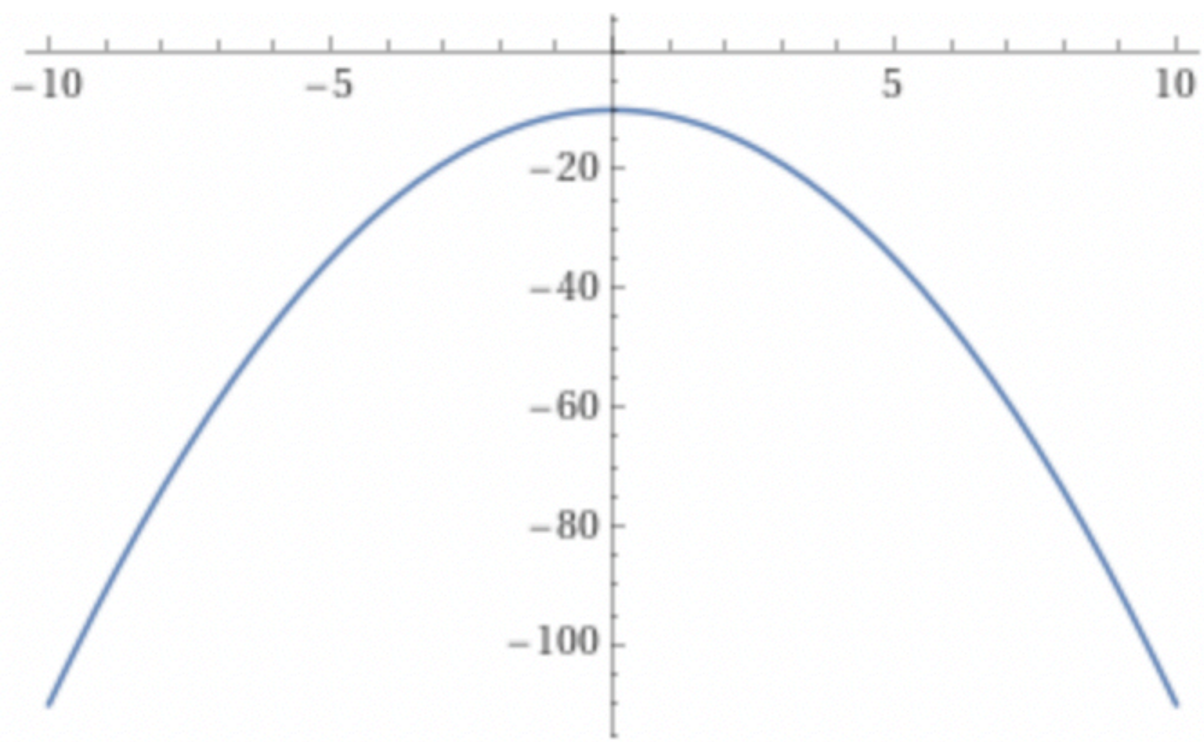


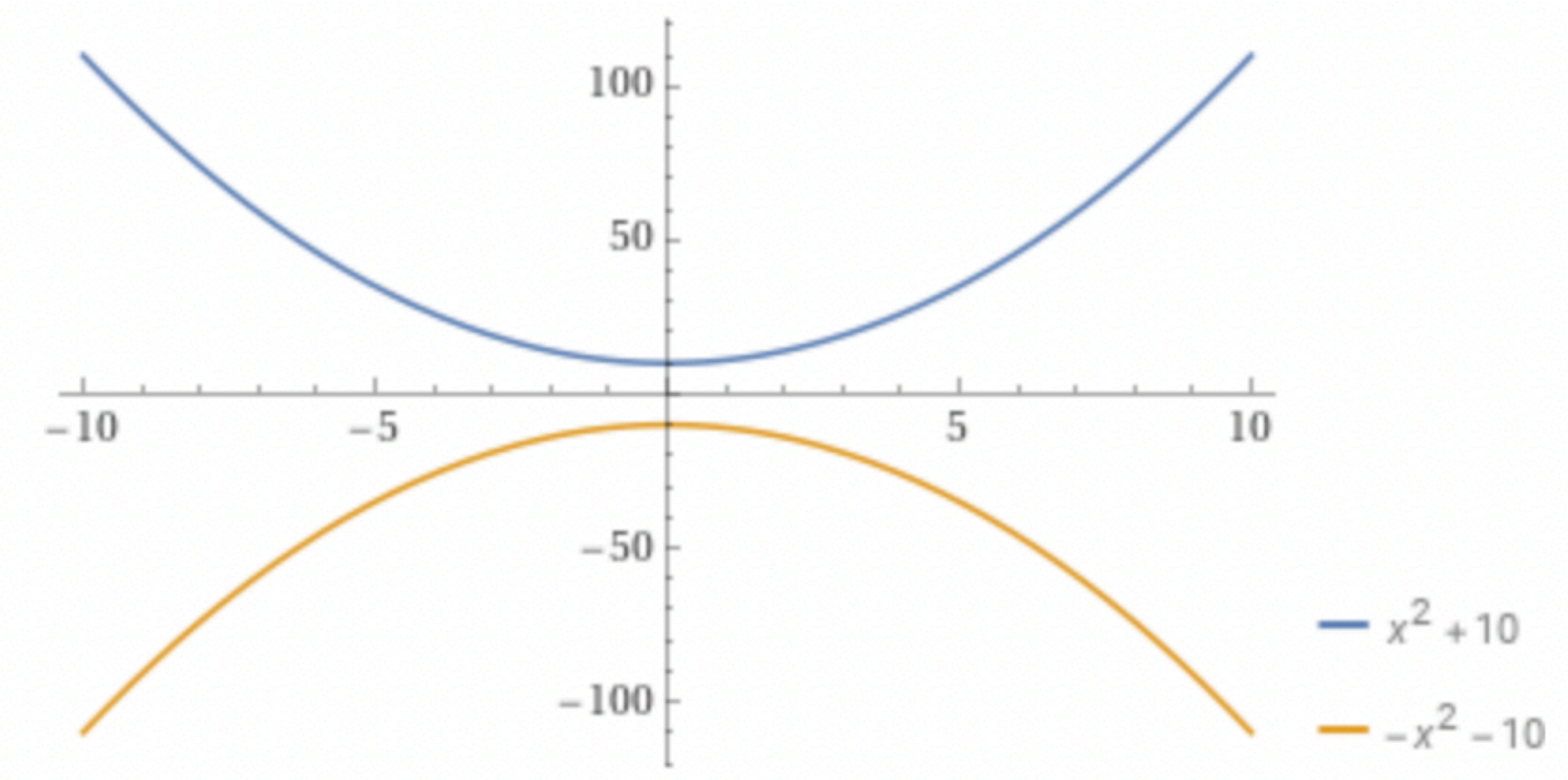
- To get graph of $f + g$
  - Take the graphs of f and g
  - For each x, add the corresponding y coordinates



How to get mirror image of the function? Treat x-axis as the mirror


100
80
60
40
20
−10
−5
5
10


−10
−5
5
10
−20
−40
−60
−80
−100

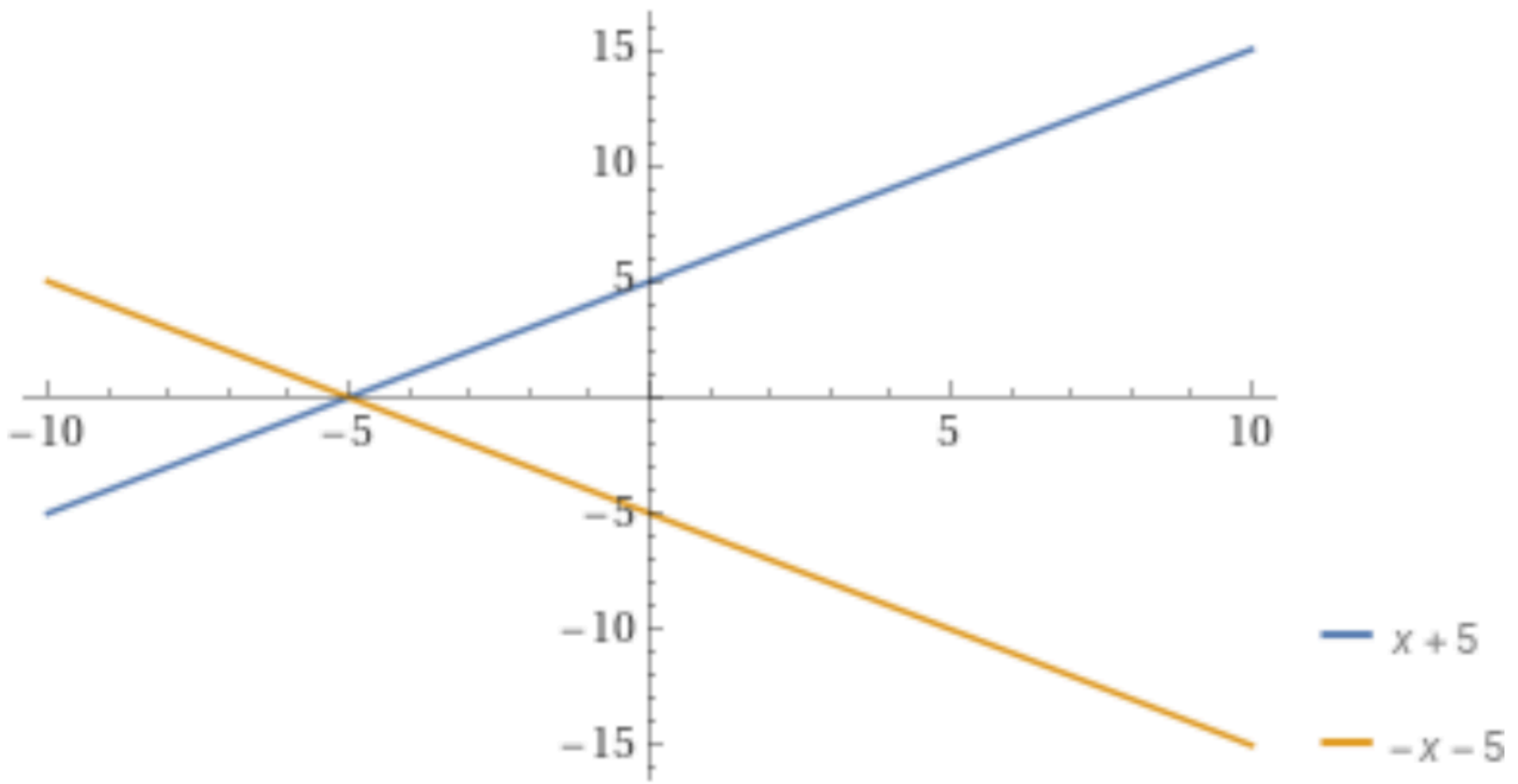


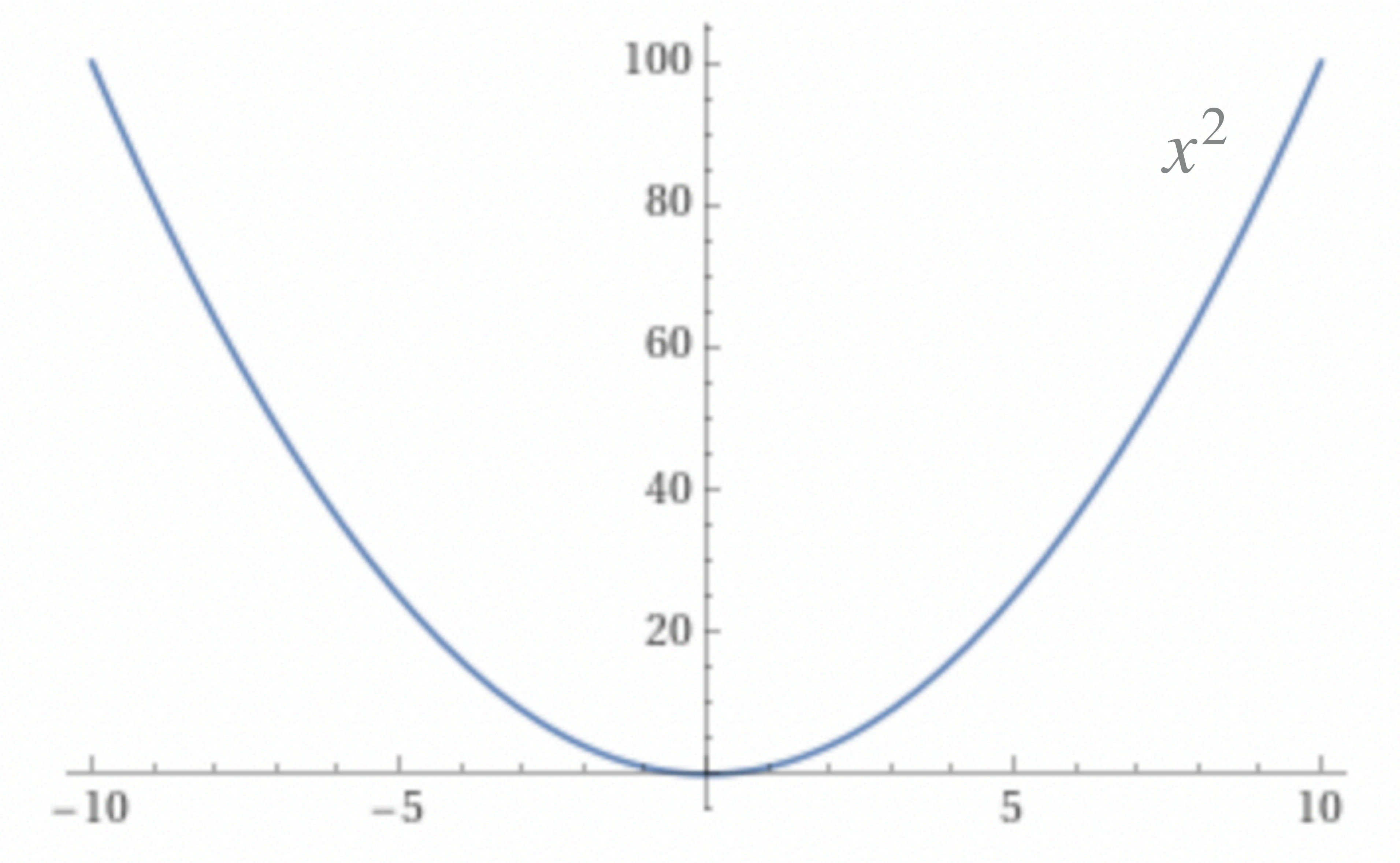
Reflect f: $g(x) = -f(x)$

- New function obtained from f
  - For each x, multiply $f(x)$ by -1

COMBINING FUNCTIONS

- For functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, define two new functions
  - Sum $(f + g)(x) = f(x) + g(x)$
  - Difference $(f - g)(x) = f(x) - g(x)$
  - Product $(f \cdot g)(x) = f(x) \cdot g(x)$
  - *Quotient* $(f/g)(x) = f(x)/g(x)$ *except where* $g(x) = 0$

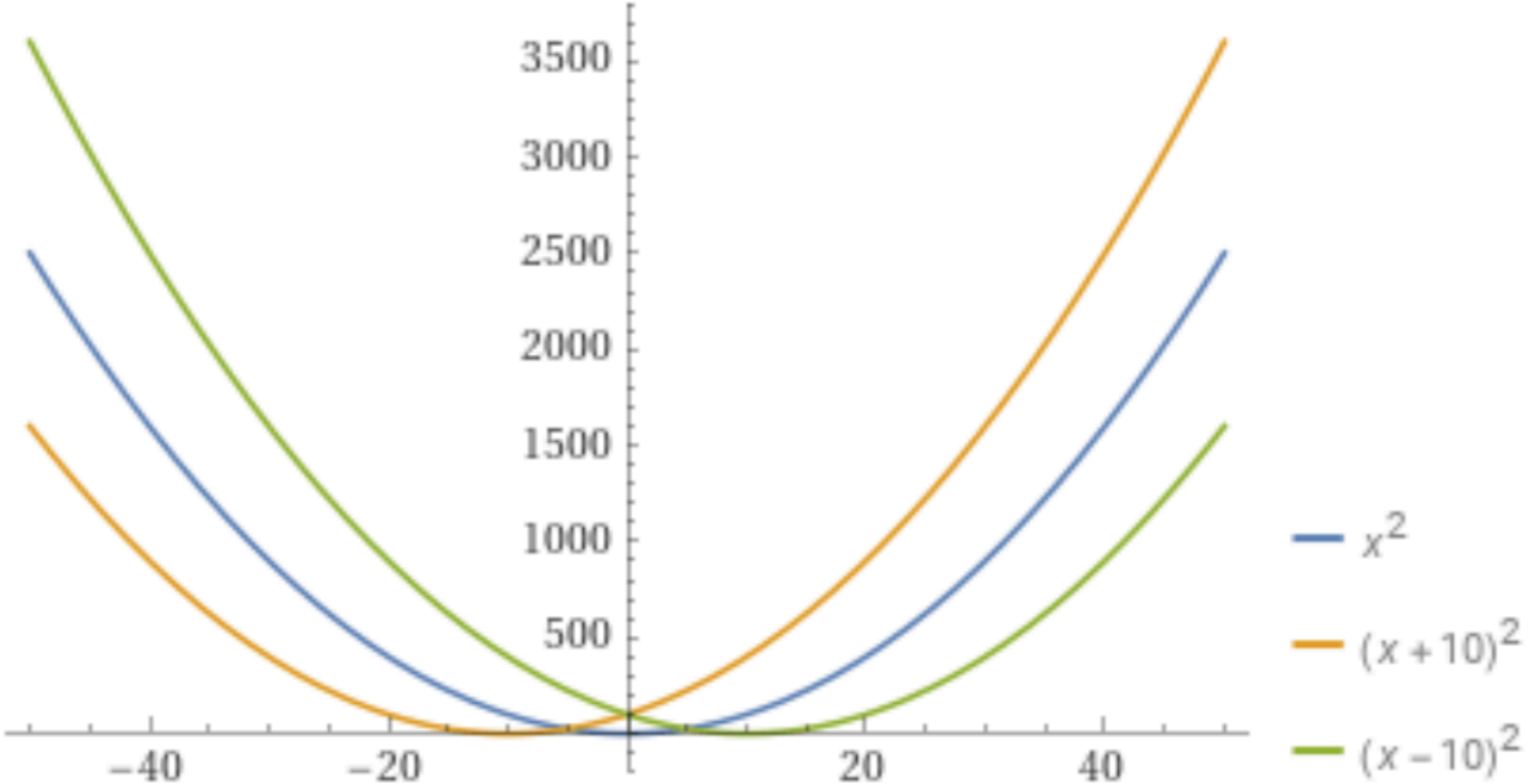




How to move the function left or right?

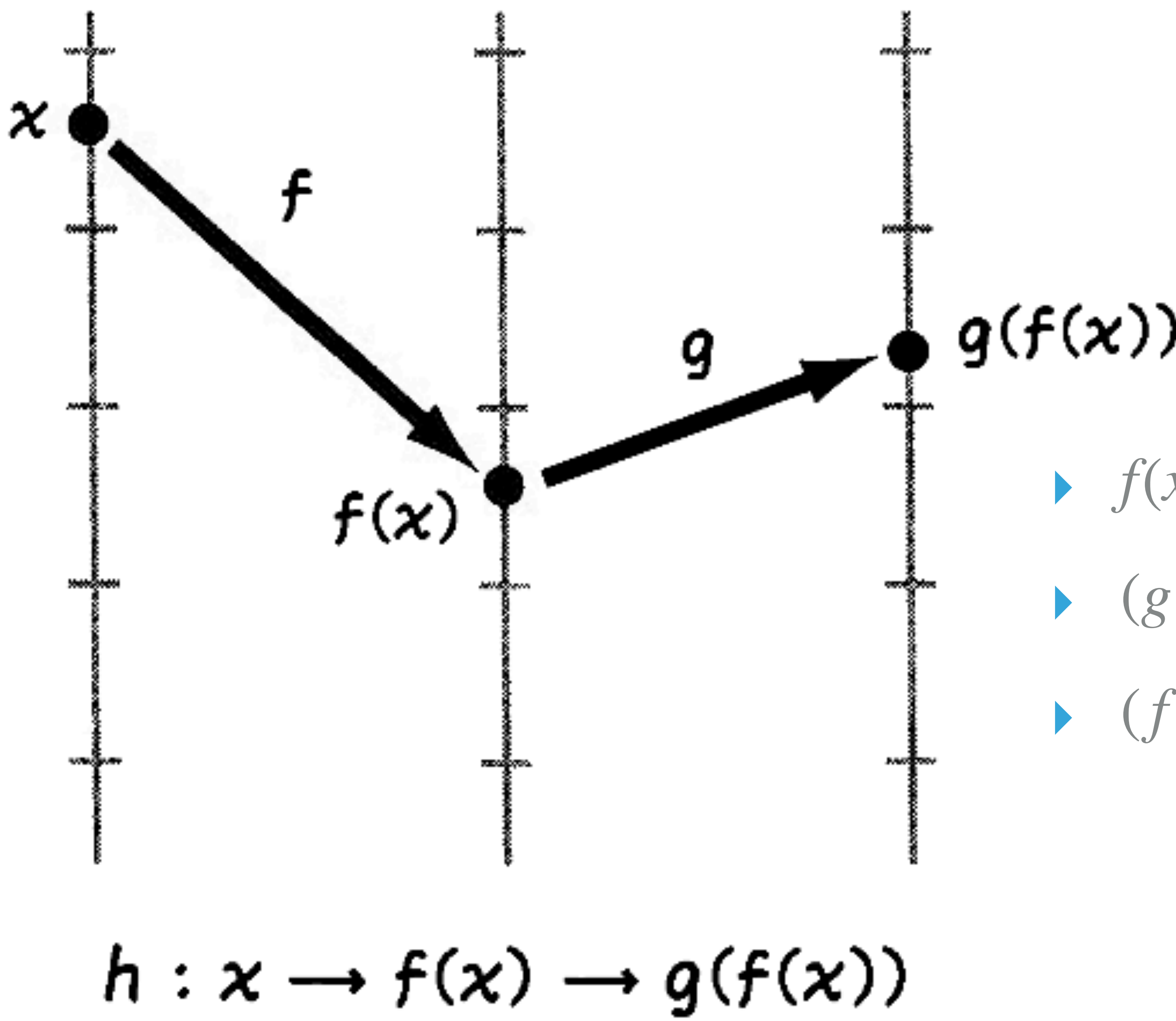
Change the point of intersection of function with x-axis

TRANSFORMATION – HORIZONTAL TRANSLATION



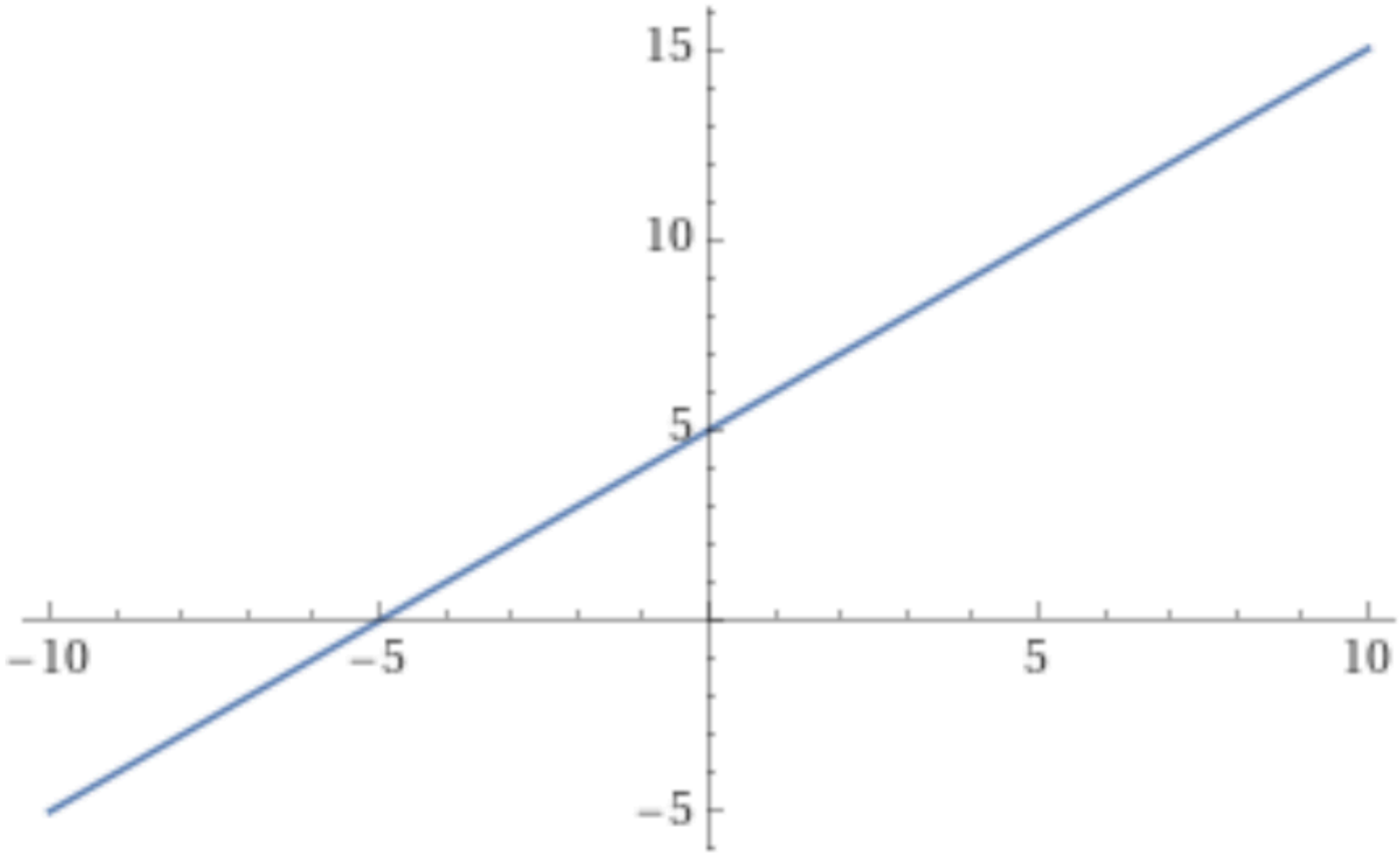
COMPOSING FUNCTIONS

- For functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$, define a new function
  - $g \circ f : \mathbb{R} \to \mathbb{R}$ where $(g \circ f)(x) = g(f(x))$ for each $x \in \mathbb{R}$



- $f(x) = x^2, g(x) = x + 5$
- $(g \circ f)(x) = g(f(x)) = x^2 + 5$
- $(f \circ g)(x) = f(g(x)) = (x + 5)^2$

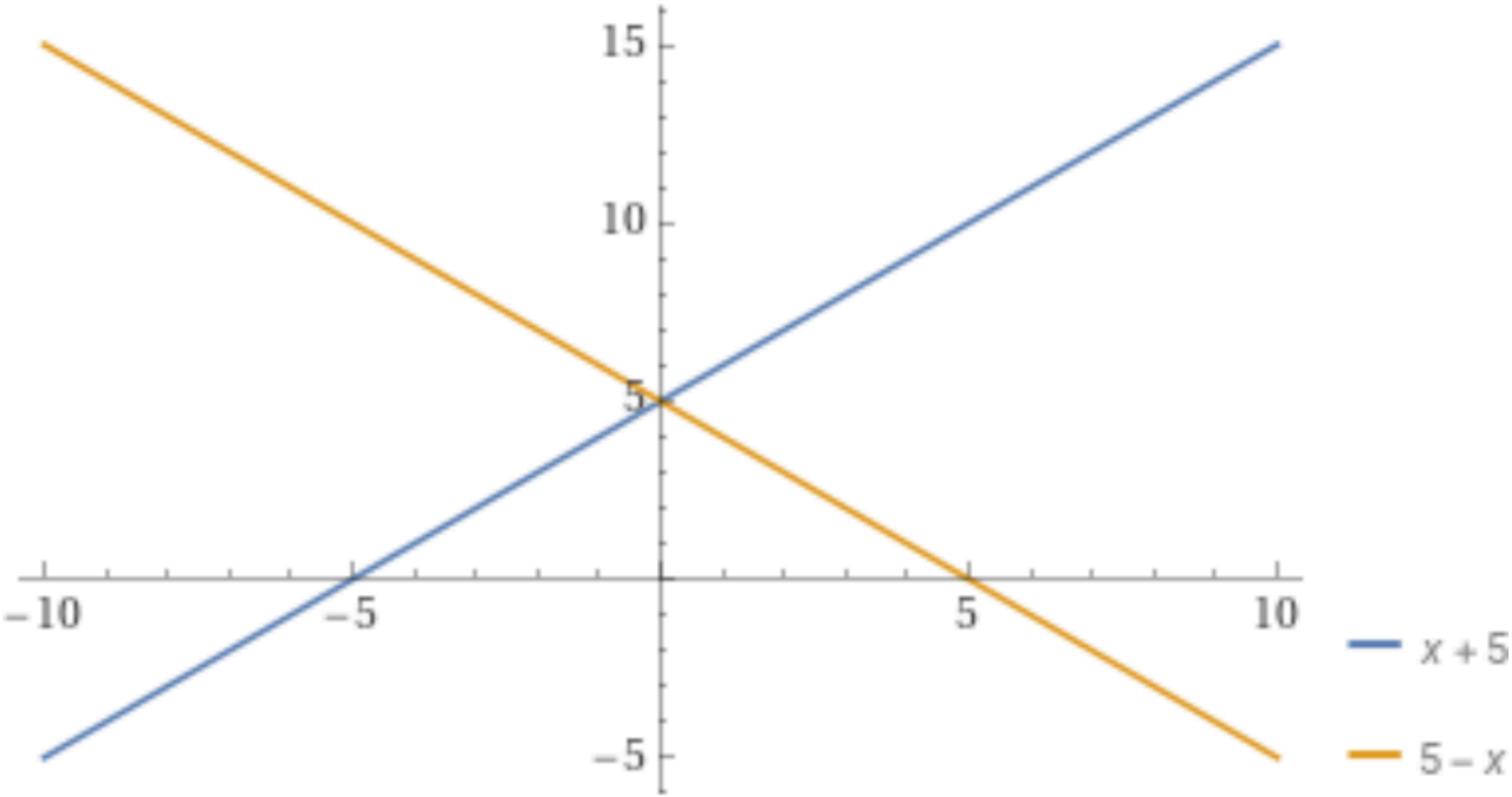
TRANSFORMATION OF FUNCTIONS

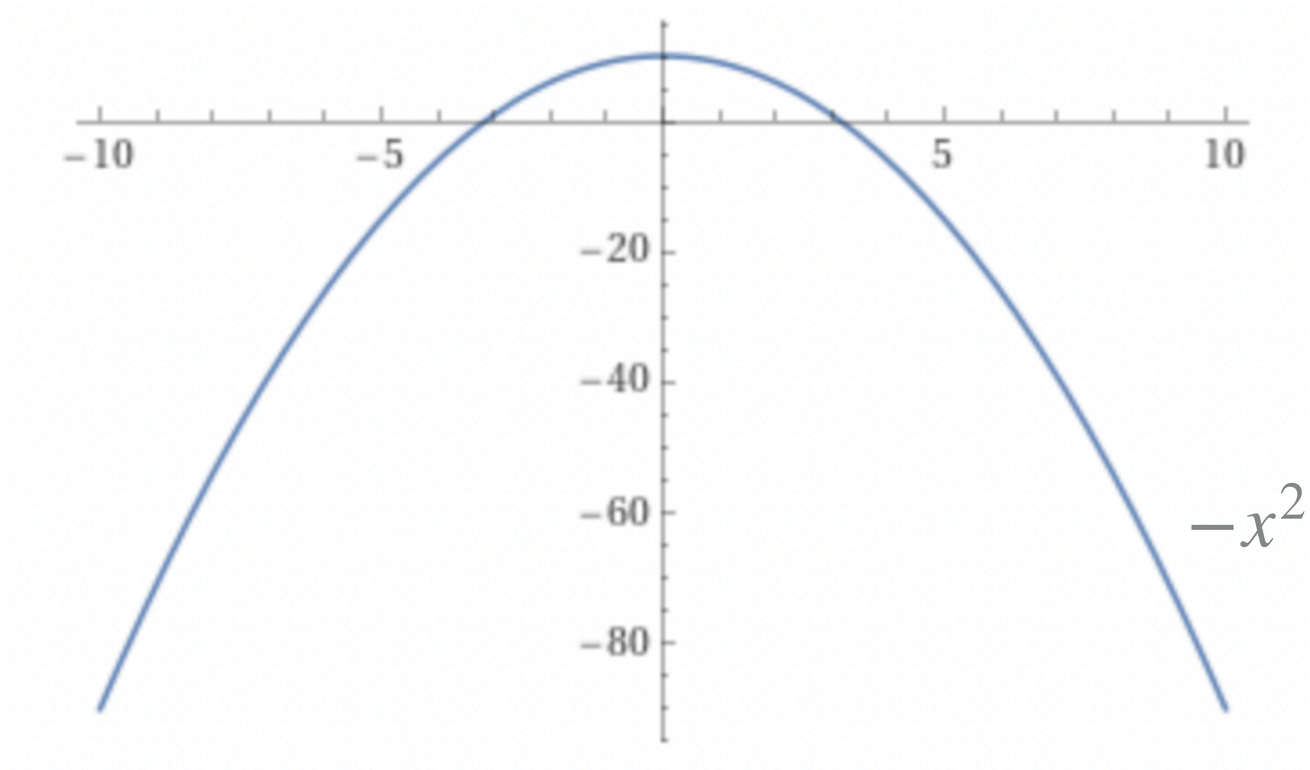


$x + 5$

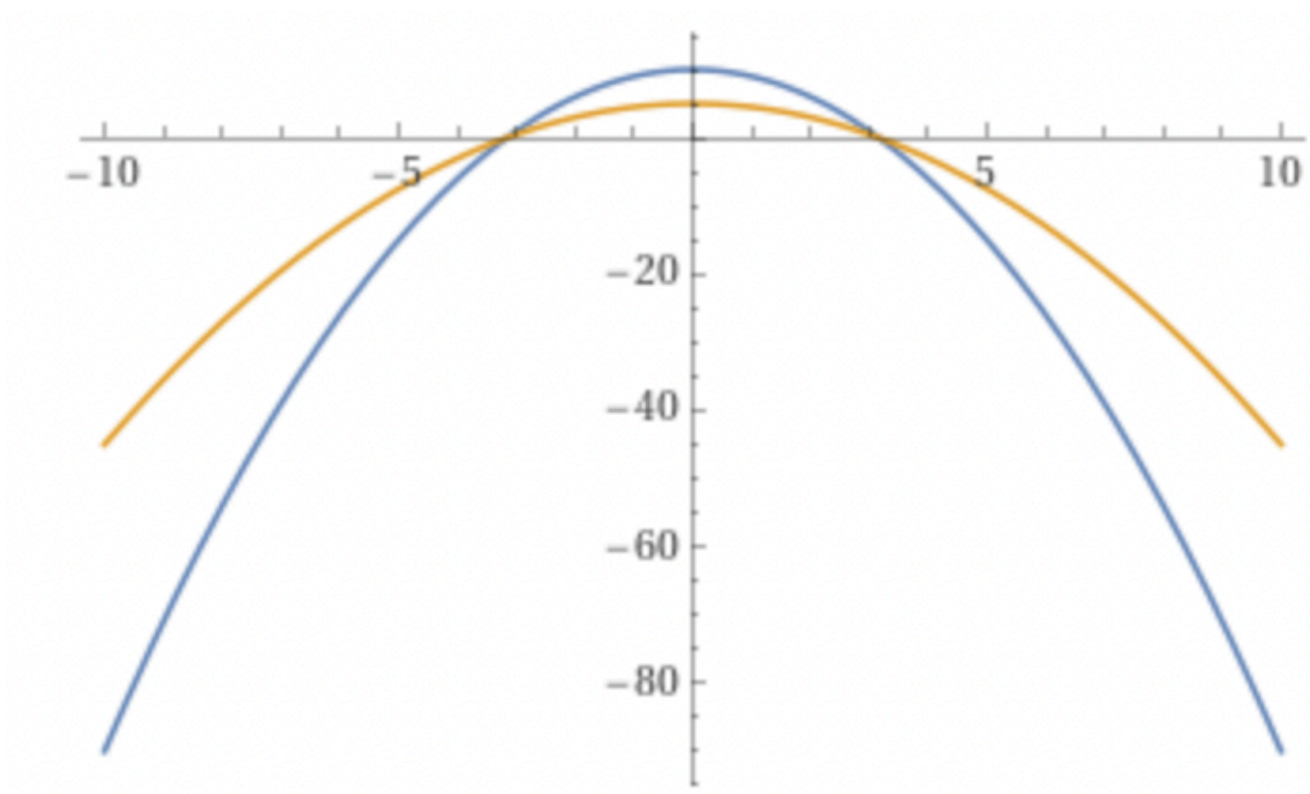
How to get mirror image of the function? Treat y-axis as the mirror

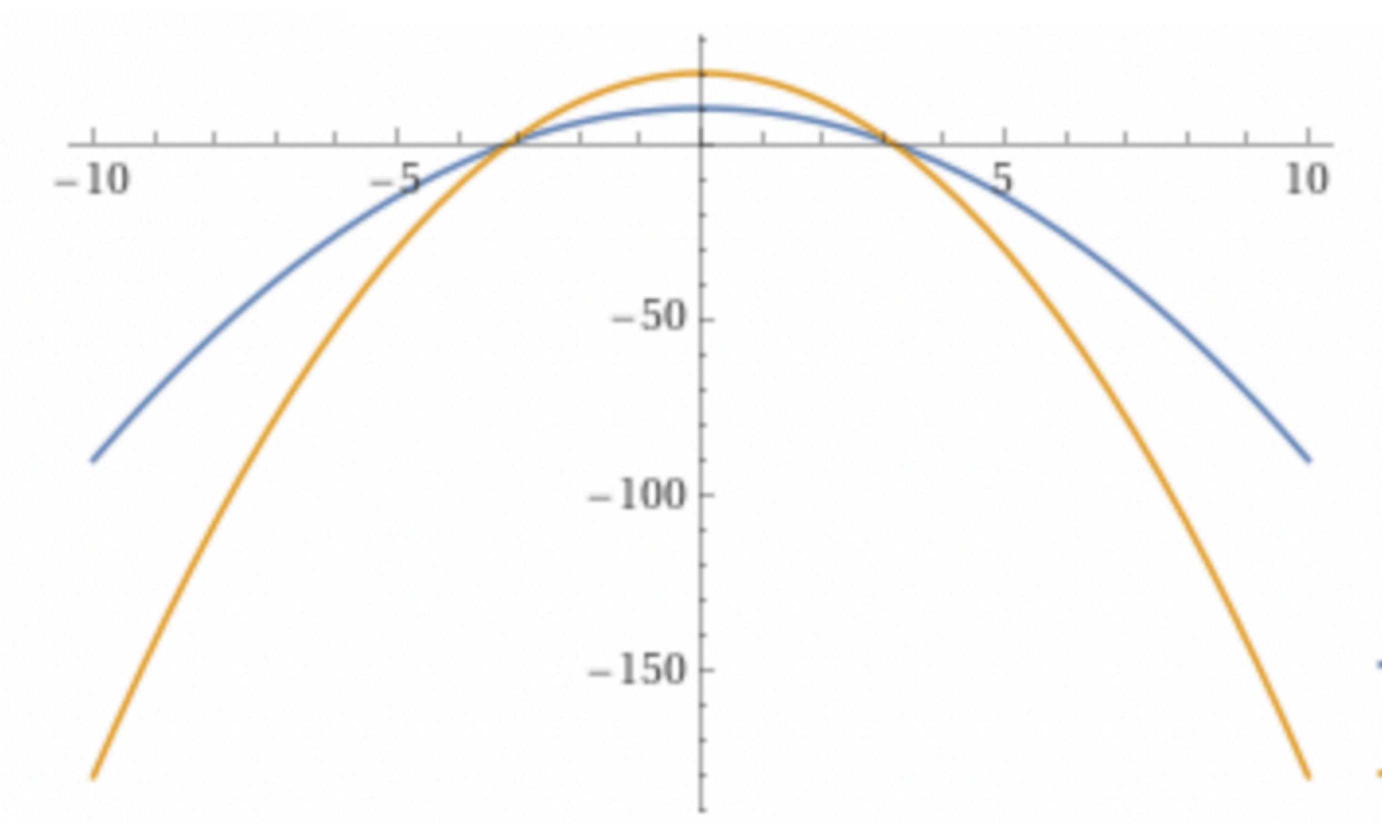
TRANSFORMATION – REFLECT ABOUT Y-AXIS





$-x^2 + 10$

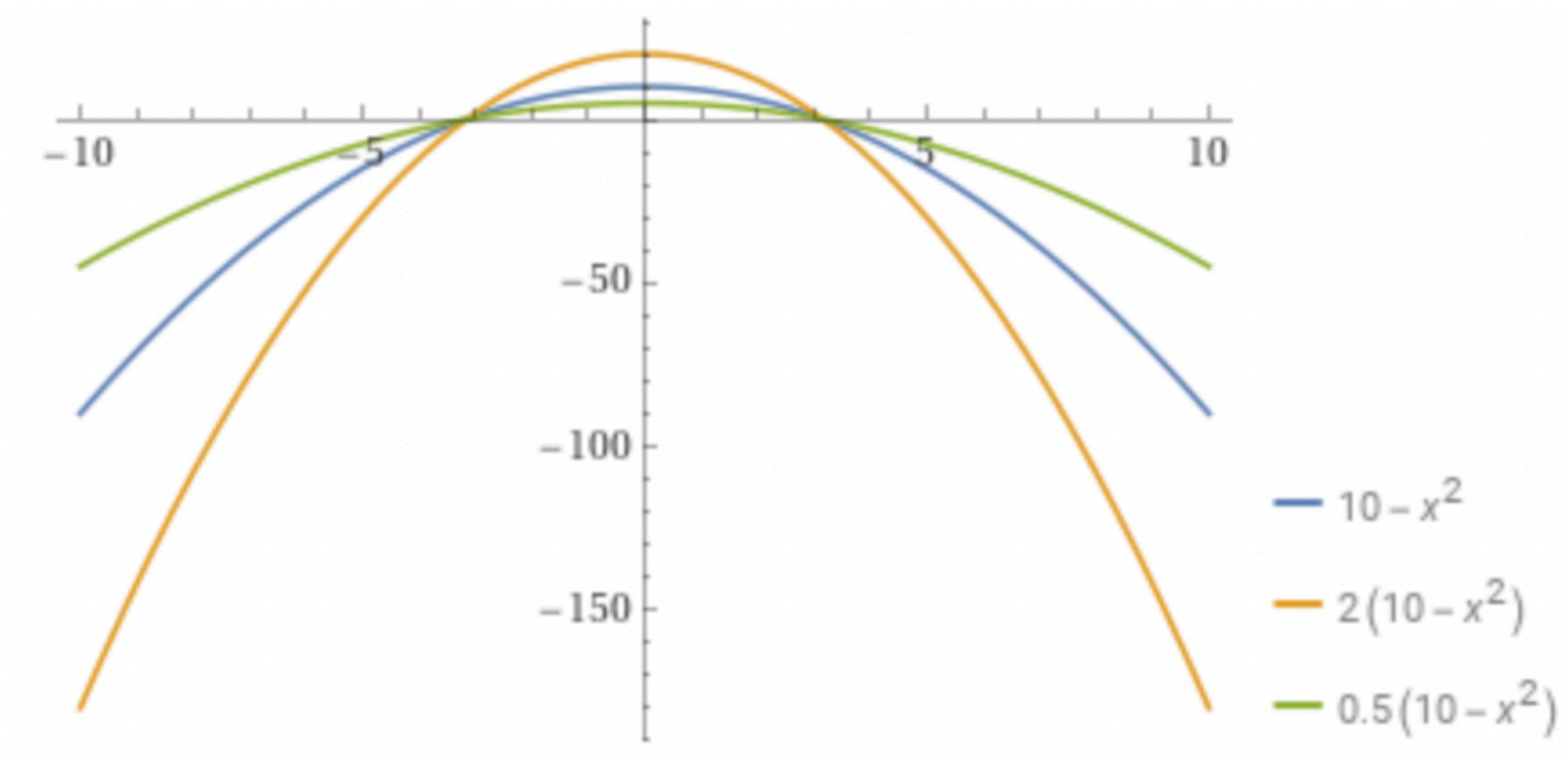


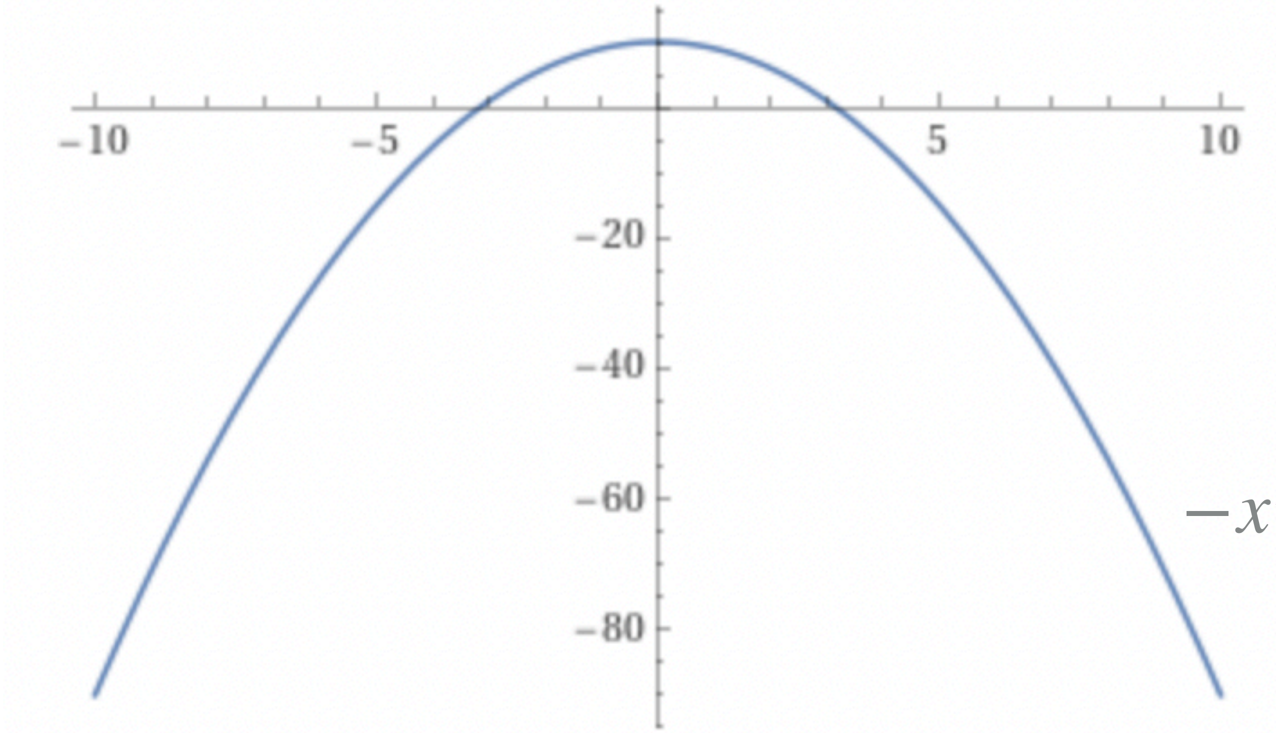


Compress the function vertically

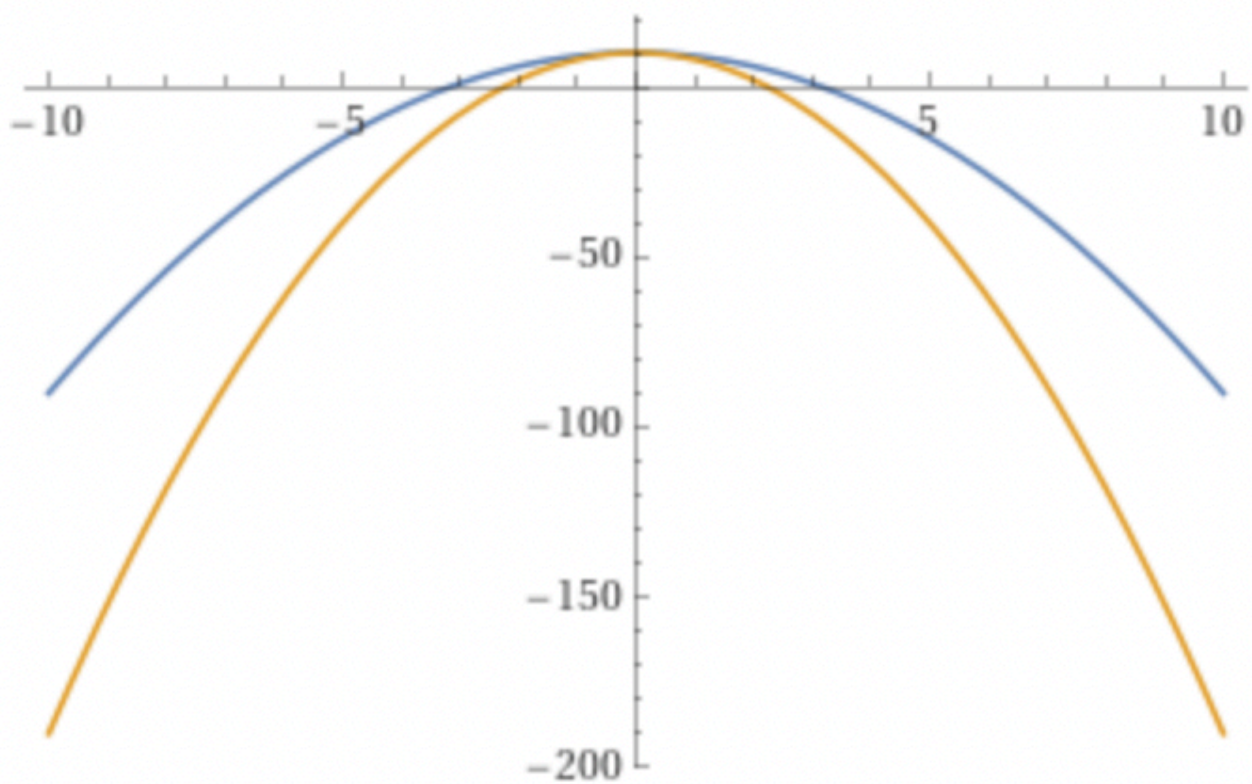
Stretch the function vertically

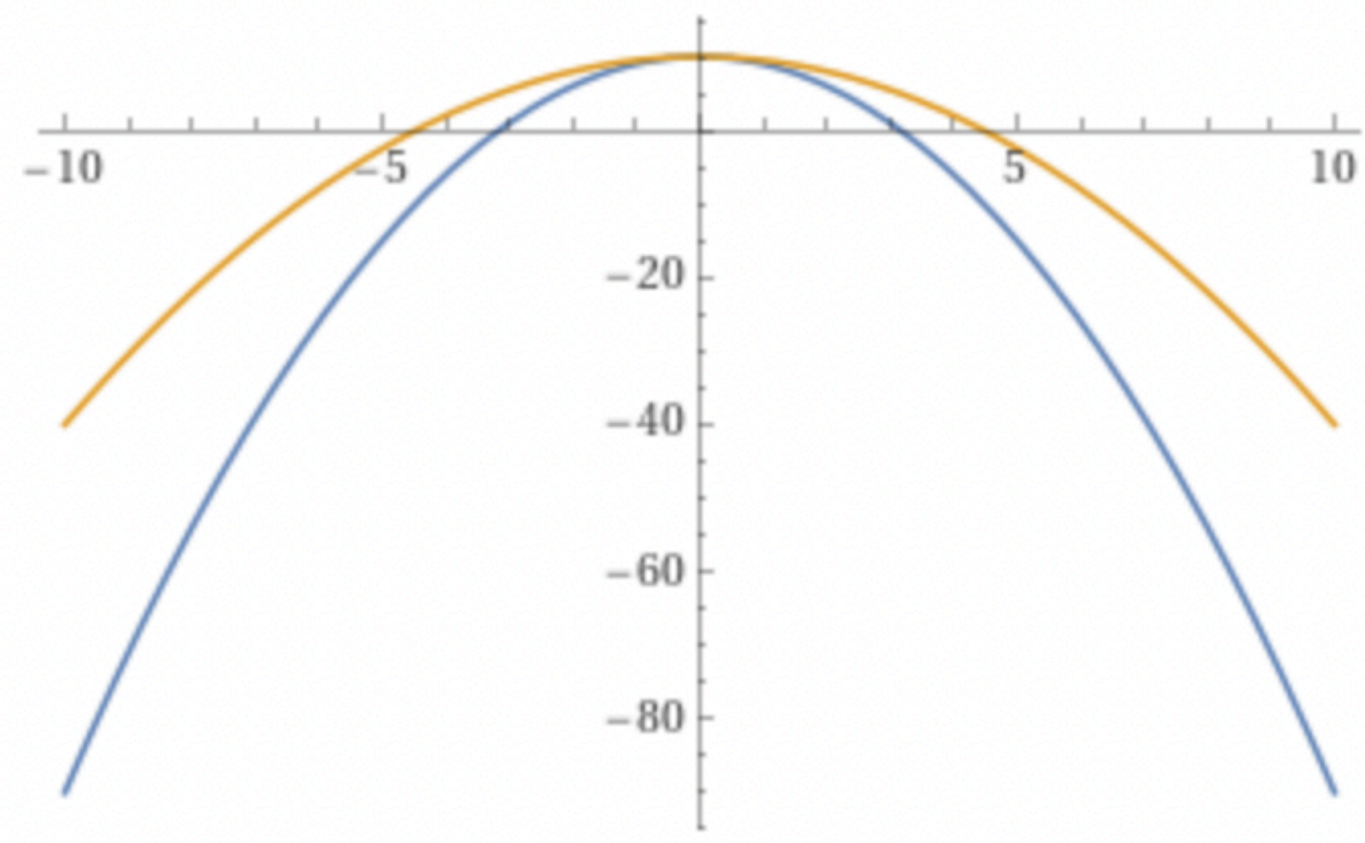
TRANSFORMATION – VERTICAL STRETCH/COMPRESS





$-x^2 + 10$

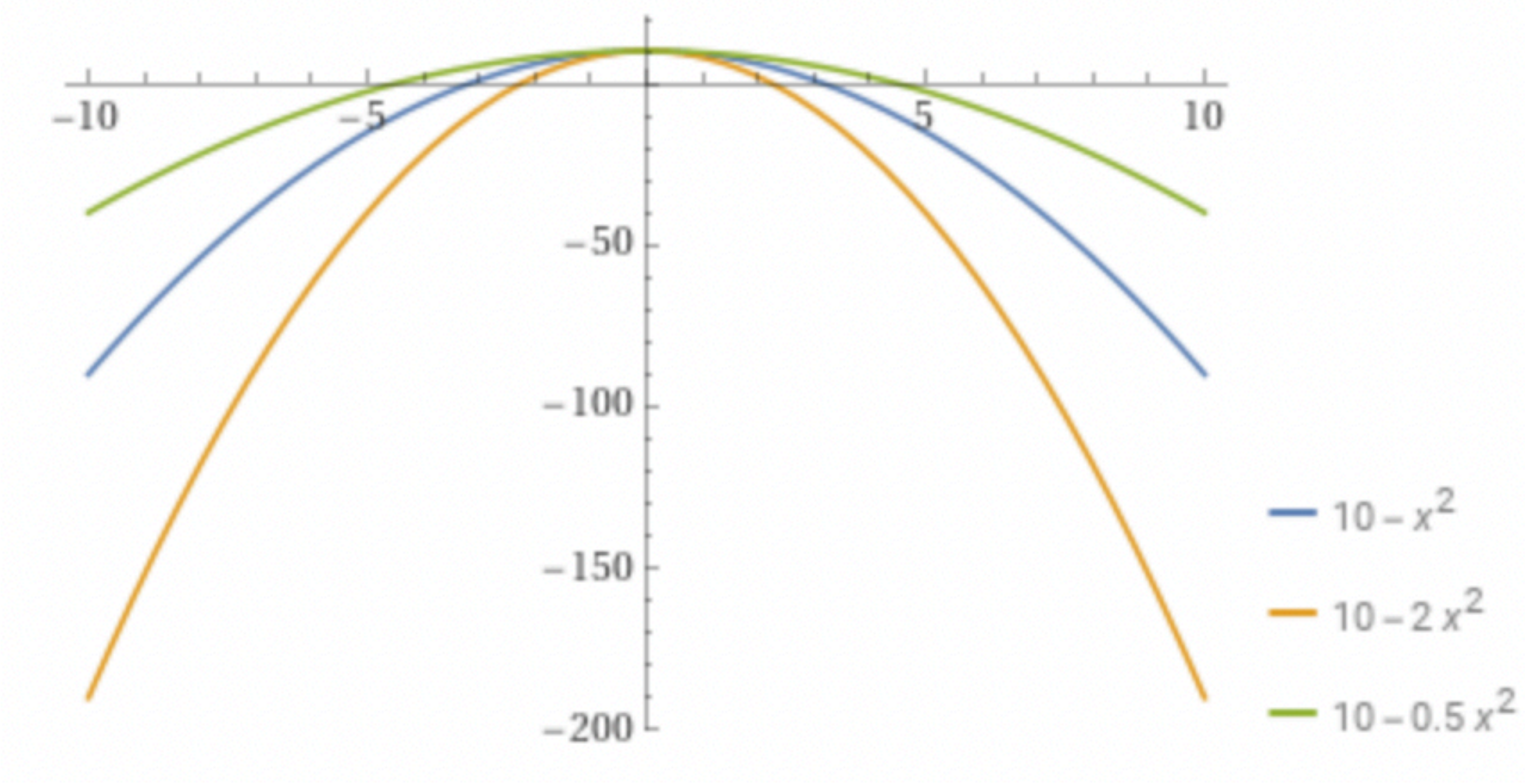




Compress the function horizontally

Stretch the function horizontally

TRANSFORMATION – HORIZONTAL SCALING

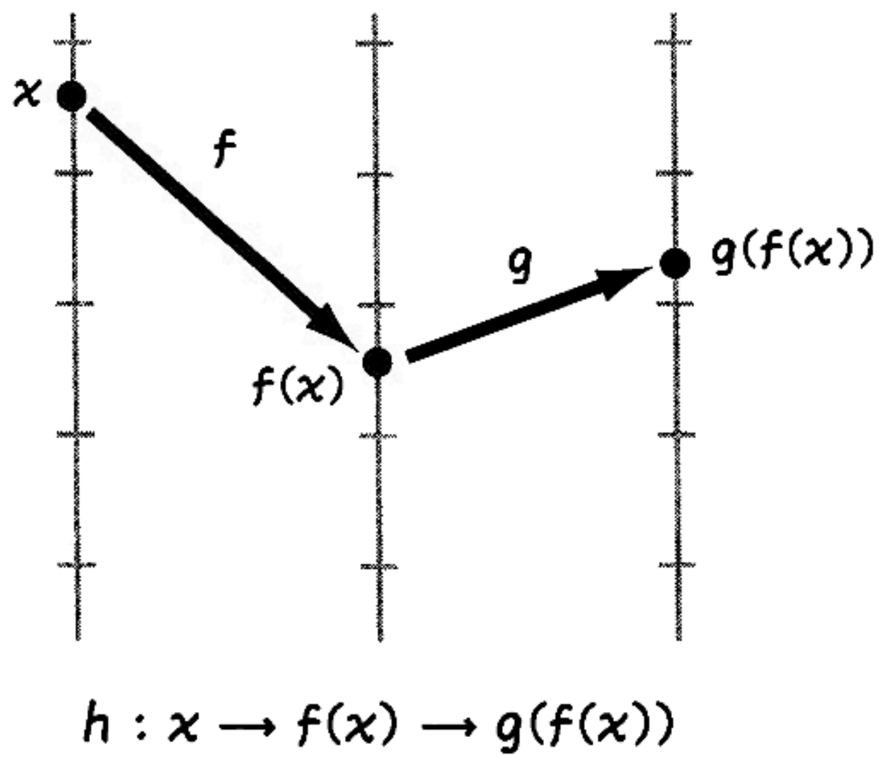


SPECIAL FUNCTION TRANSFORMATIONS

Translation	Horizontal	$f(x+k)$	$k>0$ move left, $k<0$ move right
	Vertical	$f(x)+k$	$k>0$ move up, $k<0$ move down
Reflection	x-axis	$-f(x)$	
	y-axis	$f(-x)$	
Scaling	Horizontal	$f(kx)$	$k>0$ shrink $0<k<1$ stretch
	Vertical	$kf(x)$	

COMBINING FUNCTIONS

- For functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$, define new functions
  - Sum $(f + g)(x) = f(x) + g(x)$
  - Difference $(f - g)(x) = f(x) - g(x)$
  - Product $(f \cdot g)(x) = f(x) \cdot g(x)$
  - *Quotient* $(f/g)(x) = f(x)/g(x)$ *except where* $g(x) = 0$
  - *Composition* $g \circ f : \mathbb{R} \to \mathbb{R}$ *where* $(g \circ f)(x) = g(f(x))$



- $f(x) = x^2, g(x) = x + 5$
- $(g \circ f)(x) = g(f(x)) = x^2 + 5$
- $(f \circ g)(x) = f(g(x)) = (x + 5)^2$