Worksheet- Sequences **Name:**
23/11/2025

Consider the sequence (F_n) defined by $F_1 = 1, F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$. This sequence is known as the Fibonacci sequence - it is a sequence in which each element is the sum of the two elements that precede it.

1. Write down the first 10 terms of the sequence (F_n).
2. Verify that $F_n F_{n+2} - F_{n+1}^2 = (-1)^{n+1}$ for $n = 1, 2$ and 3. Can we prove this for every natural number n?
3. Let (a_n) be a new sequence defined by $a_n = \frac{F_{n+1}}{F_n}$ for $n \geq 1$. Write down the first 6 terms of the sequence (a_n).
4. Prove that the sequence (a_n) is bounded.
5. Prove that the sequence $a_2, a_4, a_6, a_8, \ldots$ ((a_{2n}), where $a_{2n} = \frac{F_{2n+1}}{F_{2n}}$) is decreasing. Note that this implies the sequence (a_{2n}) converges as it is also bounded.
6. Let $l = \lim_{n\to\infty} a_{2n}$. Prove that $l = \frac{1+\sqrt{5}}{2}$. Hint: observe that $l = \lim_{n\to\infty} \frac{F_{2n+1}}{F_{2n}} = \lim_{n\to\infty} \frac{F_{2n}}{F_{2n-1}}$.
7. Prove that the sequence $a_1, a_3, a_5, a_7, \ldots$ (denoted as (a_{2n-1})) is increasing and converges to $\frac{1+\sqrt{5}}{2}$.
8. Does the sequence (a_n) converge ? If so, to what limit?

The number $\frac{1+\sqrt{5}}{2}$ is known as the golden ratio. Two quantities $a > b > 0$ are in golden ratio if $\frac{a+b}{a} = \frac{a}{b}$.